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Study at Two Dimensions of Thermal Transfer through a Fibers Panel Subjected to Climatic Constraints in Dynamic Frequency Regulations Established

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Abstract

From resolution of two-dimensional equation of heat in dynamic frequency regime, we have plotted evolution curves of temperature according to depth of material or in lateral direction. They will allow us to evaluate thermal behavior of towed material. Aim of study is to use fibers as a thermal insulating material by proposing a method for determining effective thermal insulation layer in dynamic frequency regime.

Keywords

Thermal Transfer, Heat Exchange Coefficient, Frequency Dynamic Regime, Fibers

1. Introduction

Controlling energy consumption in homes during hot weather, or in cold rooms requires, among other things, good thermal insulation. Problems of cost of synthetic materials and end-of-life management lead us to propose use of biodegradable materials. Thus study of thermal behavior of material for use in thermal insulation [1] [2] is of major interest in search for energy control.

Several methods for calculating transient or established dynamic regimes [3] [4] are developed for simulation of thermal behavior within material.

We propose in this study resolution in dynamic regime established by imposing boundary conditions particular to panel of fibers.

Temperature curves along material depth or in lateral direction will allow us to evaluate thermal behavior of fibers material. Influence of the thermal exchange coefficient on front face will be highlighted.

2. Theory

Equation of heat transfer through rectangular material to absence of internal source, is written in two dimensions (**Figure 1**):

$$\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\frac{\partial T}{\partial t} \quad (1)$$

α is thermal diffusivity coefficient of the material ($m^2 \cdot s^{-1}$) and is expressed as

$$\alpha=\frac{\lambda}{\rho C} \quad (2)$$

where:

- ρ ($kg \cdot m^{-3}$)is density of material,
- C ($J \cdot kg^{-1} \cdot K^{-1}$) is mass thermal capacity,
- λ ($W \cdot m^{-1} \cdot K^{-1}$) is thermal conductivity of material.

For method by separating variables, we propose:

$$T(x,y,t)=X(x)\cdot Y(y)\cdot Z(t) \quad (3)$$

(3) in (1) leads to following expressions:

$$\alpha\left(\frac{1}{X(x)}\cdot\frac{\partial^2 X(x)}{\partial x^2}+\frac{1}{Y(y)}\cdot\frac{\partial^2 Y(y)}{\partial y^2}\right)=\frac{1}{Z(t)}\cdot\frac{\partial Z(t)}{\partial t} \quad (4)$$

$$\begin{cases}\dfrac{1}{Z(t)}\cdot\dfrac{\partial Z(t)}{\partial t}=i\cdot\omega & (5)\\[2ex] \dfrac{\partial^2 X(x)}{\partial x^2}+\beta^2\cdot X(x)=0 & (6)\\[2ex] \dfrac{\partial^2 Y(y)}{\partial y^2}+\mu^2\cdot Y(y)=0 & (7)\end{cases}$$

Equations (5)-(7) are deduced from following relationships (8):

$$\omega=i\alpha\left(\beta^2+\mu^2\right) \quad \text{with} \quad \mu=i\sqrt{\frac{i\cdot\omega}{\alpha}+\beta^2} \quad (8)$$

Solutions of preceding equations give:

$$Z(t)=a\cdot e^{i\cdot\omega\cdot t} \quad (9)$$

$$X(x)=a_1\cos\beta\cdot x+b_1\sin\beta\cdot x \quad (10)$$

$$Y(y)=a_2\cos\mu\cdot y+b_2\sin\mu\cdot y \quad (11)$$

which gives general expression of temperature through material in form:

$$T(x,y,t)=\left(a_1\cos\beta\cdot x+b_1\sin\beta\cdot x\right)\left(a_2\cos\mu\cdot y+b_2\sin\mu\cdot y\right)\left(e^{i\cdot\omega\cdot t}\right) \quad (12)$$

We apply to system boundary conditions, Equations (13)-(16), reflecting heat exchanges between faces of material and external environment:

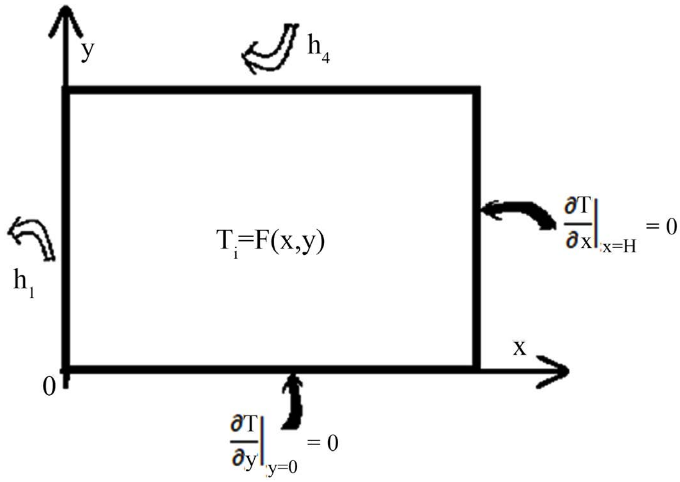

Figure 1. Study model.

$$\begin{cases} \left.\dfrac{\partial T}{\partial x}\right|_{x=0} = H_1\left(T - T_i\right) & (13) \\ \left.\dfrac{\partial T}{\partial x}\right|_{x=H} = 0 & (14) \\ \left.\dfrac{\partial T}{\partial y}\right|_{y=0} = 0 & (15) \\ \left.\dfrac{\partial T}{\partial y}\right|_{y=L} = -H_4 \cdot T & (16) \end{cases}$$

with:

$$T_i = T_0 \cdot e^{i \cdot \omega \cdot t} \tag{17}$$

$$H_1 = \frac{h_1}{\lambda} \quad \text{and} \quad H_4 = \frac{h_4}{\lambda} \tag{18}$$

T_i is the temperature imposed on the front face.

We pose:

$$X_0 = T_0 = k \cdot a_1 \tag{19}$$

$$\beta \cdot b_1 = H_1\left(a_1 - X_0\right) \tag{20}$$

we obtain:

$$\frac{b_1}{a_1} = \frac{H_1}{\beta}(1 - k) \tag{21}$$

$$\tan \beta \cdot H = \frac{H_1}{\beta}(1 - k) \tag{22}$$

$$\tan \mu \cdot L = \frac{H_4}{\mu} \tag{23}$$

Graphical representations of **Figure 2** and **Figure 3** correspond to transcendental Equation (22) and Equation (23) for obtaining eigenvalues β_m and μ_n respectively and given in **Table 1**.

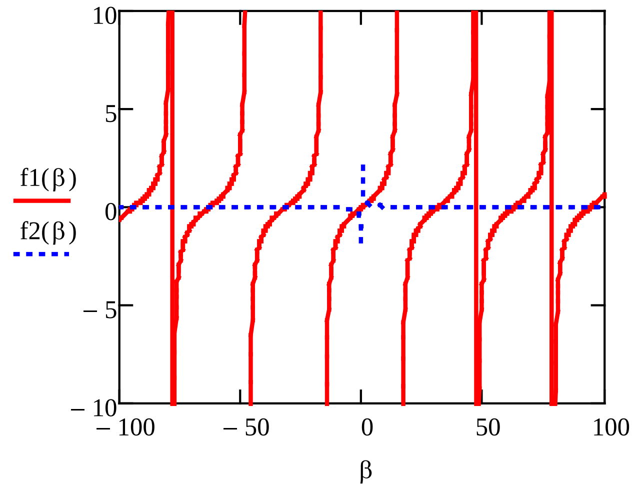

Figure 2. Eigenvalues β_m.

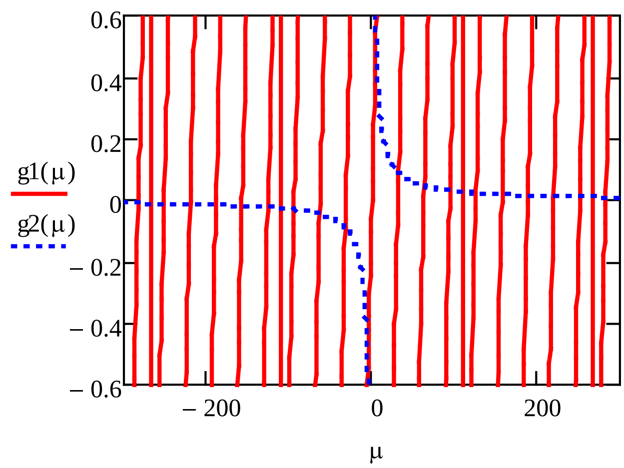

Figure 3. Eigenvalues μ_n.

Table 1. Positives eingenvalues: $h_1 = h_4 = 0.5\ \text{W}\cdot\text{m}^{-2}\cdot{}^\circ\text{C}^{-1}$.

n	1	2	3	4	5	6	7	8	9	0	11
μ_n, β_n (rad·m^{-1})	5	32	65	65	96	127	158	192	219	250	284

Solution of equation of heat can thus be written:

$$T(x,y,t) = \sum_m \sum_n A_{mn} \left(\beta_m \cos \beta_m \cdot x + \left(H_1(1-k)\right) \sin \beta_m \cdot x\right)\left(\cos \mu_n \cdot y\right) \cdot \mathrm{e}^{\omega_{mn} \cdot t} \quad (24)$$

To explain the coefficient A_{mn}, we pose:

$$F(x,y) = \sum_m \sum_n A_{mn} \left(\beta_m \cos \beta_m \cdot x + \left(H_1(1-k)\right) \sin \beta_m \cdot x\right)\left(\cos \mu_n \cdot y\right) \quad (25)$$

It can be written as:

$$F(x,y) = \sum_m B_m \cdot \left(\beta_m \cos \beta_m \cdot x + \left(H_1(1-k)\right) \sin \beta_m \cdot x\right) \quad (26)$$

with:

$$B_m = \sum_{n=1}^{\infty} A_{mn} \left(\cos \mu_n \cdot y\right) \quad (27)$$

$$B_m = \frac{1}{\int_0^H \left(\beta_m \cos\beta_m \cdot x + \left(H_1\left(1-k\right)\right)\sin\beta_m \cdot x\right)^2 \mathrm{d}x} \cdot \int_0^H F\left(x,y\right)\cdot\left(\beta_m \cos\beta_m \cdot x + \left(H_1\left(1-k\right)\right)\sin\beta_m \cdot x\right)\mathrm{d}x \tag{28}$$

$$A_{mn} = \frac{1}{\int_0^L \left(\cos\mu_n \cdot y\right)^2 \mathrm{d}y}\int_0^L B_m \cdot \cos\mu_n \cdot y\mathrm{d}y \tag{29}$$

We replace B_m in (38) by its expression obtained by (37), we obtain:

$$A_{mn} = \frac{1}{\int_0^H\int_0^L \left(\beta_m \cos\beta_m \cdot x + \left(H_1\left(1-k\right)\right)\sin\beta_m \cdot x\right)^2 \left(\cos\mu_n \cdot y\right)^2 \mathrm{d}x\mathrm{d}y} \cdot \int_0^H\int_0^L F\left(x,y\right)\cdot\left(\beta_m \cos\beta_m \cdot x + \left(H_1\left(1-k\right)\right)\sin\beta_m \cdot x\right)\left(\cos\mu_n \cdot y\right)\mathrm{d}x\mathrm{d}y \tag{30}$$

We apply normalization condition (31):

$$\begin{cases} N\left(\beta_m\right) = \int_0^H \left(\beta_m \cos\beta_m \cdot x + \left(H_1\left(1-k\right)\right)\sin\beta_m \cdot x\right)^2 \mathrm{d}x \\ N\left(\mu_n\right) = \int_0^L \left(\cos\mu_n \cdot y\right)^2 \mathrm{d}y \end{cases} \tag{31}$$

$$N\left(\beta_m\right) = \frac{1}{2}H\left(\beta_m + \left(H_1\left(1-k\right)^2 + H_1\left(1-k\right)\right)\right) \tag{32}$$

$$N\left(\mu_n\right) = \frac{1}{2}\left(\frac{H_4}{\left(\mu_n^2 + H_4^2\right)} + L\right) \tag{33}$$

We finally get:

$$A_{mn} = \frac{4}{\left(H\left(\beta_m + \left(H_1\left(1-k\right)^2 + H_1\left(1-k\right)\right)\right)\right)\left(\frac{H_4}{\left(\mu_n^2 + H_4^2\right)} + L\right)} \cdot \int_0^H\int_0^L F\left(x,y\right)\cdot\left(\beta_m \cos\beta_m \cdot x + \left(H_1\left(1-k\right)\right)\sin\beta_m \cdot x\right)\left(\cos\mu_n \cdot y\right)\mathrm{d}x\mathrm{d}y \tag{34}$$

Coefficient k is relative to the thermal effisitivity, it defines transmission of heat to wall of material: $0 \le k \le 1$.

$k = 0$: we have perfect thermal insulation behavior on surface of material;

$k = 1$: we have perfect conductor behavior on surface of material.

We worked with $k = 0.85$.

Taking into account results obtained in previous studies [4] [5], we consider an average value of coefficient translating heat exchanges on surface of material: $k = 0.85$.

3. Results and Discussion

Solution (25) of heat equation made it possible to draw simulation curves of **Figures 4-7** showing evolution of temperature inside material.

Figure 4 shows evolution of temperature according depth inside material in frequency modulation. We have an accumulation of heat resulting in heating of material for $x < 0.03$ m there is no heat exchange on the lower face $y = 0$. For $x > 0.03$ m, we have decrease in amplitude of temperature which tends at end of

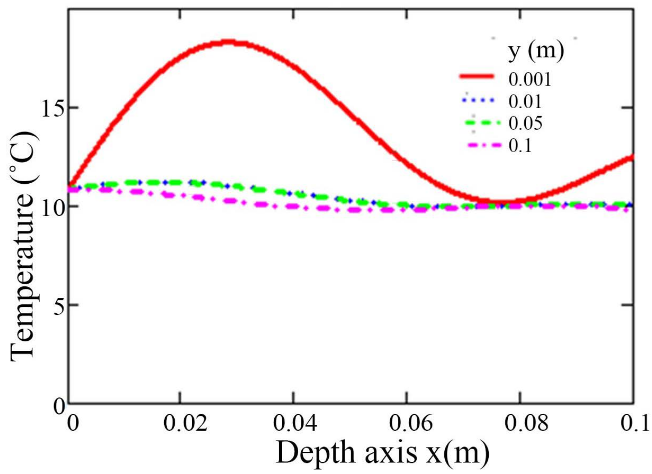

Figure 4. Evolution of temperature according to depth. $T_i = 10˚C$, $T_e = 35˚C$, $\lambda = 0.174$ $W \cdot m^{-1} \cdot ˚C^{-1}$, $h_1 = h_4 = 0.5$ $W \cdot m^{-2} \cdot ˚C^{-1}$.

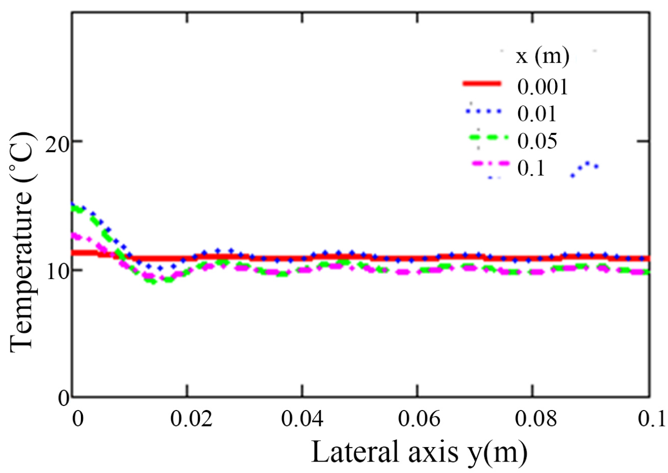

Figure 5. Evolution of temperature along lateral axis. $T_i = 10˚C$, $T_e = 35˚C$, $\lambda = 0.174$ $W \cdot m^{-1} \cdot ˚C^{-1}$, $h_1 = h_4 = 0.5$ $W \cdot m^{-2} \cdot ˚C^{-1}$.

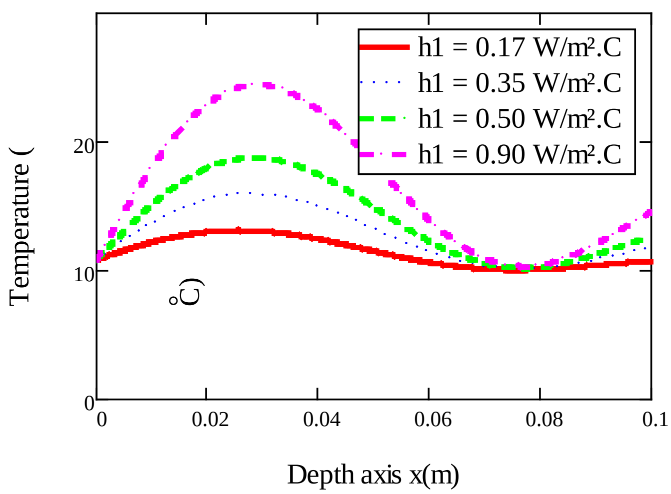

Figure 6. Evolution of temperature according to depth. Influence of heat exchange coefficient. $T_i = 10˚C$, $T_e = 35˚C$, $\lambda = 0.174$ $W \cdot m^{-1} \cdot ˚C^{-1}$, $h_4 = 0.5$ $W \cdot m^{-2} \cdot ˚C^{-1}$.

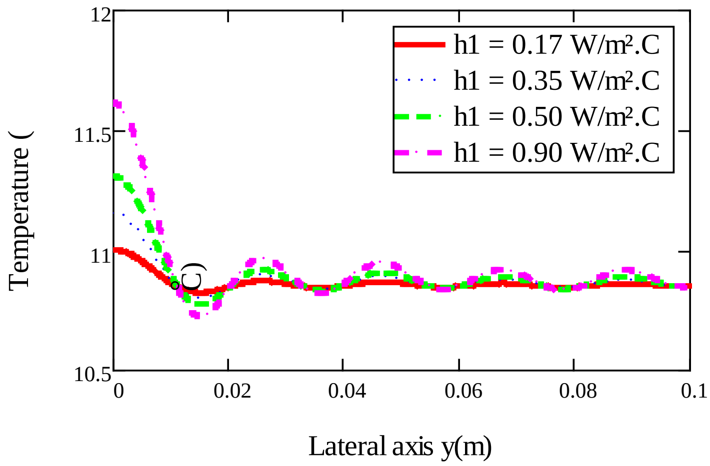

Figure 7. Evolution of temperature along lateral axis. Influence of heat exchange coefficient. $T_i = 10˚C$, $T_e = 35˚C$, $\lambda = 0.174\ W \cdot m^{-1} \cdot ˚C^{-1}$, $h_4 = 0.5\ W \cdot m^{-2} \cdot ˚C^{-1}$.

excitation towards amplitude of initial temperature of material; material warms up slightly.

Figure 5 shows evolution of temperature along lateral axis. Overheating phenomenon in vicinity of $y = 0$ is confirmed because there is no heat exchange with respect to outside. Beyond $y = 0.02$ m, material temperature fluctuates around 10˚C, which corresponds to low overheating of material.

Figure 6 and Figure 7 show influence of heat exchange coefficient at front face on temperature. Exchange coefficient contributes significantly to heating of wall and propagation of heat along axis of depths. On other hand, along lateral axis, this influence is practically nil.

4. Validation

Methods of thermal characterization in numerical simulation [6] or analytical [7] [8] present results of evolutions of temperature and density of heat flux comparable through different materials.

5. Conclusion

Starting from resolution of equation of two-dimensional heat in frequency dynamic regime, we obtained curves which show thermal behavior of fibers material subjected to different climatic constraints. Thermal insulating nature of material is highlighted by an evanescent tendency of external excitation inside material.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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