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Full Length Research Paper

Flow induced by non-coaxial rotations of porous disk and a fluid in a porous medium

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This paper deals with the flow of an incompressible, viscous and electrically conducting fluid in a porous medium when no slip condition is no longer valid. The fluid is bounded by a non-conducting porous disk. The flow is due to non-coaxial rotations of porous disk and a fluid at infinity. The fluid is electrically conducting in the presence of a constant applied magnetic field in the transverse direction. Analytic treatment for the arising problem is made for velocity components. An analytical solution of the problem is developed using Laplace transform method. A critical assessment is made for the case of partial slip and no-slip conditions and discussed these results in medium which is porous. Graphical results of velocity components are presented for various values of pertinent dimensionless parameters.

Key words: Viscous fluid, non-coaxial rotations, Laplace transform porous medium, partial slip condition.

INTRODUCTION

The equations which govern the flow of a Newtonian fluid are the Navier-Stokes equations. These equations are highly non-linear partial differential equations and known exact solutions are few in number. Exact solutions are very important not only because they are solutions of some fundamental flows but also because they serve as accuracy checks for experimental, numerical and asymptotic methods. Although computer techniques make the complete numerical integration of the Navier-Stokes equations feasible, the accuracy of the result can be established by a comparison with an exact solution. Exact solutions of the Navier-Stokes equations have been given by many authors, Berker (1963), Siddiquiet al. (2001), Erdogan (2000), Murthy and Ram (1978) and Rao and Kasiviswanathan (1987) in various situations. In continuation, Erdogan (2000) studied the flow due to non-coaxial rotations of a disk oscillating in its own plane and fluid at infinity. Hayat et al. (2001) discussed the magnetohydrodynamic (MHD) flow of a viscous fluid due to non-coaxial rotations of a porous disk and a fluid at infinity. Rajagopal (1992) presented a review for flows of Newtonian and non-Newtonian fluids between parallel disks rotating about a common axis.

However, no attempt has been made to discuss the MHD flow due to non-coaxial rotations of porous disk and a fluid through a porous medium when no slip condition is no longer valid. The aim of this paper is to present such attempt. Infect, the objective of the present work is to extend the analysis of reference, Hayatet al. (20017) in order to study the effects of partial slip on the flow in a porous medium. The disk and the fluid exhibit non-coaxial rotations. By using the Laplace transform method, the structures of the associated boundary layers are investigated in a porous medium when no-slip condition is no longer valid. The fluids exhibiting slip are very important from technological point of view. For example, the polishing of artificial heart valves and internal cavities in a variety of manufactured parts is achieved by imbedding such fluids with abrasives. The influence of various emerging parameters is discussed with the help of graphs.

Mathematical description of the problem

We consider an incompressible electrically conducting viscous fluid occupying the porous space $z > 0$. The fluid is in contact with a porous disk occupying the position $z = 0$. The disk and fluid are in a state of non-coaxial rotation with l being the distance between the axes of the rotations of the disk and fluid at infinity. A uniform magnetic field B_0 is applied in the transverse direction to flow. The magnetic Reynolds number is very small and thus the induced magnetic field is neglected. The electric field is taken zero. Initially the disk and fluid at infinity are rotating about z'-axis with constant angular velocity Ω. For $t = 0$, the disk starts to rotate impulsively about the z -axis and the fluid at infinity rotates about z'-axis with the same angular velocity Ω. For the flow under consideration, the boundary and initial considerations are;

$$\begin{aligned} &\mathrm{u}-\beta\frac{\partial \mathrm{u}}{\partial \mathrm{z}} = -\Omega \mathrm{y}, \quad \mathrm{v}-\beta\frac{\partial \mathrm{v}}{\partial \mathrm{z}} = \Omega \mathrm{x} \quad \text{at} \quad \mathrm{z}=0, \quad \mathrm{t}>0, \\ &\mathrm{u} = -\Omega(\mathrm{y}-l), \quad \mathrm{v} = \Omega \mathrm{x} \quad \text{as} \quad \mathrm{z}\to\infty \quad \text{for all} \quad \mathrm{t}, \\ &\mathrm{u} = -\Omega(\mathrm{y}-l), \quad \mathrm{v} = \Omega \mathrm{x} \quad \text{at} \quad \mathrm{t} = 0, \quad \mathrm{z} > 0, \end{aligned} \tag{1}$$

Where β is slip coefficient and u and v are the x - and y -components of velocity defined by;

$$\mathrm{u} = -\Omega \mathrm{y}+\mathrm{f}(\mathrm{z},\, \mathrm{t}), \quad \mathrm{v} = \Omega \mathrm{x}+\mathrm{g}(\mathrm{z},\, \mathrm{t}). \tag{2}$$

Equation 2 together with continuity equation gives;

$$\mathrm{w} = -\mathrm{W}_0, \tag{3}$$

in which $W_0 > 0$ and $W_0 < 0$ respectively indicates the suction and blowing velocities.

After eliminating the pressure gradient, the momentum equation along with Equations (2) and (3) gives;

$$\mathrm{v}\frac{\partial^3 w}{\partial z^3}+\mathrm{W}_0\frac{\partial^2 w}{\partial z^2}-\frac{\partial^2 w}{\partial t \partial z}-(\mathrm{i}\Omega+\frac{\sigma}{\rho}\mathrm{B}_0{}^2+\frac{\phi}{k}\upsilon)\frac{\partial w}{\partial z}=0, \tag{4}$$

Where σ is the electrical conductivity of fluid, ρ is the density, υ is the kinematic viscosity, ϕ is the porosity, k is the permeability and;

$$\mathrm{W} = \mathrm{f}+\mathrm{i}\,\mathrm{g}. \tag{5}$$

The conditions in Equation (1) now become;

$$\begin{aligned} &\mathrm{W}(0,\mathrm{t}) = \beta\frac{\partial \mathrm{w}}{\partial \mathrm{z}} \quad \text{at} \quad \mathrm{z} = 0, \quad \mathrm{t} > 0, \\ &\mathrm{W}(\infty,\mathrm{t}) = \Omega l \quad \text{at} \quad \mathrm{z}\to\infty \quad \text{for all} \quad \mathrm{t}, \\ &\mathrm{W}(z,0) = \Omega l \quad \text{at} \quad \mathrm{t} = 0, \quad \mathrm{z} > 0. \end{aligned} \tag{6}$$

Defining

$$\mathrm{G} = \frac{\mathrm{w}}{\Omega l}, \quad \xi = \sqrt{\frac{\Omega}{2\mathrm{v}}}\mathrm{z}, \quad \tau=\Omega \mathrm{t}, \tag{7}$$

In Equations (4) and (6) and then solving the resulting problem by Laplace transform procedure we obtain;

$$G(\xi,\tau)=1-e^{-\frac{s}{2}\xi}\left[\begin{array}{l} \frac{1}{a^2-A}\left(\frac{1}{\sqrt{\pi\tau}}e^{-\frac{\xi^2}{4\tau}-A\tau}-ae^{\xi a+a^2\tau-A\tau}erfc\left(\frac{\xi}{2\sqrt{\tau}}+a\sqrt{\tau}\right)\right) \\ +\frac{1}{2A-2a\sqrt{A}}\left(\frac{1}{\sqrt{\pi\tau}}e^{-\frac{\xi^2}{4\tau}-A\tau}-\sqrt{A}e^{\xi\sqrt{A}}erfc\left(\frac{\xi}{2\sqrt{\tau}}+\sqrt{A\tau}\right)\right)+ \\ \frac{1}{2A+2a\sqrt{A}}\left(\frac{1}{\sqrt{\pi\tau}}e^{-\frac{\xi^2}{4\tau}-A\tau}+\sqrt{A}e^{-\xi\sqrt{A}}erfc\left(\frac{\xi}{2\sqrt{\tau}}-\sqrt{A\tau}\right)\right) \end{array}\right], \tag{8}$$

Where;

$$\begin{aligned} &\mathrm{A} = \left(\frac{\mathrm{s}}{2}\right)^2+\mathrm{N}+\mathrm{i}+\lambda, \quad \mathrm{a} = 1+\frac{\beta \mathrm{S}}{2}, \quad \mathrm{S} = \frac{\mathrm{w}_0}{\sqrt{2\Omega \mathrm{v}}}, \\ &\mathrm{N} = \frac{\sigma \mathrm{B}_0{}^2}{\rho\Omega}, \quad \lambda = \frac{\phi\upsilon}{\mathrm{k}\Omega}, \quad \beta = \beta\sqrt{\frac{\Omega}{2\upsilon}}, \end{aligned} \tag{9}$$

and erfc (·) is the complementary error function. It should be noted that for $\lambda = \beta = 0$, we get the results of Hayat et al. (2001). Equation (8) for large time reduces.

$$G(\xi,\tau)=\left[\frac{1+\sqrt{A}e^{-\xi\sqrt{A}-\frac{s}{2}}}{a\sqrt{A}+A}\right]$$

$$-e^{-\frac{s}{2}\xi}\left[\begin{array}{l} \frac{1}{a^2-A}\left(\frac{1}{\sqrt{\pi\tau}}e^{-\frac{\xi^2}{4\tau}-A\tau}-ae^{\xi a+a^2\tau-A\tau}erfc\left(\frac{\xi}{2\sqrt{\tau}}+a\sqrt{\tau}\right)\right) \\ +\frac{1}{2A-2a\sqrt{A}}\left(\frac{1}{\sqrt{\pi\tau}}e^{-\frac{\xi^2}{4\tau}-A\tau}-\sqrt{A}e^{\xi\sqrt{A}}erfc\left(\frac{\xi}{2\sqrt{\tau}}+\sqrt{A\tau}\right)\right)+ \\ \frac{1}{2A+2a\sqrt{A}}\left(\frac{1}{\sqrt{\pi\tau}}e^{-\frac{\xi^2}{4\tau}-A\tau}+\sqrt{A}e^{-\xi\sqrt{A}}erfc\left(\frac{\xi}{2\sqrt{\tau}}-\sqrt{A\tau}\right)\right) \end{array}\right]. \tag{10}$$

The approximate forms of the real and imaginary parts of suction solution when $\xi \ll 2\sqrt{\tau}$ and $\tau \gg 1$ are;

$$\frac{f}{\Omega 1}=1+\frac{e^{-\frac{s}{2}\xi+\xi\alpha_1}[(2\alpha_1+2+\beta S)\cos\xi\alpha_2-2\alpha_2 2\sin\xi\alpha_2]}{(2\alpha_1+2+\beta S)^2+(2\alpha_2)^2}$$
$$-\frac{e^{\left(\frac{s}{2}\xi+\alpha_2^2-\alpha_1^2\right)\tau}}{\sqrt{\pi\tau}}\frac{\left[\left(a^2+\alpha_2^2-\alpha_1^2\right)\left\{\cos 2\alpha_1 a_2\tau+2\alpha_1 a_2\sin 2\alpha_1 a_2\tau\right\}\left(1-e^{\xi a}\right)\right]}{\left(a^2+\alpha_2^2-\alpha_1^2\right)^2+\left(2\alpha_1 a_2\right)^2}$$
$$-\frac{e^{-\frac{s}{2}\xi+(a_2^2-\alpha_1^2)\tau}}{\sqrt{\pi\tau}}\frac{\left[\left(2\alpha_1^2-2\alpha_2^2-2a\alpha_1\right)\left\{\begin{matrix}\left(1-e^{\xi\alpha_1}\cos\xi\alpha_2\right)\cos 2\alpha_1 a_2\tau\\ -e^{\xi\alpha_1}\sin\xi\alpha_2\sin 2\alpha_1 a_2\tau\end{matrix}\right\}\right]}{\left(2\alpha_1^2-2\alpha_2^2-2a\alpha_1\right)^2+\left(2a\alpha_2+4\alpha_1 a_2\right)^2}$$
$$+\left(2a\alpha_2+4\alpha_1 a_2\right)\left[\left(1-e^{\xi\alpha_1}\cos\xi\alpha_2\right)\sin 2\alpha_1 a_2\tau+e^{\xi\alpha_1}\sin\xi\alpha_2\cos 2\alpha_1 a_2\tau\right]$$
$$-\frac{e^{\left(\frac{s}{2}\xi+a_2^2-\alpha_1^2\right)\tau}}{\sqrt{\pi\tau}}\left[\begin{matrix}\dfrac{\left(2\alpha_1^2-2\alpha_2^2-2a\alpha_1\right)\left\{\begin{matrix}\left(1-e^{\xi\alpha_1}\cos\xi\alpha_2\right)\cos 2\alpha_1 a_2\tau\\ +e^{-\xi\alpha_1}\sin\xi\alpha_2\sin 2\alpha_1 a_2\tau\end{matrix}\right\}}{\left(2\alpha_1^2-2\alpha_2^2-2a\alpha_1\right)^2+\left(2a\alpha_2+4\alpha_1 a_2\right)^2}+\\ \left(2a\alpha_2+4\alpha_1 a_2\right)\left(\begin{matrix}\left(e^{-\xi\alpha_1}\sin\xi\alpha_2\right)\cos 2\alpha_1 a_2\tau-\\ (1-e^{-\xi\alpha_1}\cos\xi\alpha_2)\sin 2\alpha_1 a_2\tau\end{matrix}\right)\end{matrix}\right], \quad (11)$$

$$\frac{g}{\Omega 1}=-\frac{e^{-\frac{s}{2}\xi-\xi\alpha_1}}{(2\alpha_1+2+\beta S)^2+(2\alpha_2)^2}\left[2\alpha_2\cos\xi\alpha_2+(2\alpha_1+2+\beta S)\sin\xi\alpha_2\right]$$
$$-\frac{e^{\frac{s}{2}\xi+\left(\alpha_2^2-\alpha_1^2\right)\tau}}{\sqrt{\pi\tau}}\left[\frac{2\alpha_1\alpha_2\cos 2\alpha_1\alpha_2\tau-\left(a^2+\alpha_2^2-\alpha_1^2\right)\left(1-e^{\xi a}\right)\sin 2\alpha_1\alpha_2\tau}{\left(-a^2-\alpha_2^2+\alpha_1^2\right)^2+\left(2\alpha_1\alpha_2\right)^2}\right]$$
$$-\frac{e^{\frac{s}{2}\xi-\left(\alpha_2^2-\alpha_1^2\right)\tau}}{\sqrt{\pi\tau}}\left[\begin{matrix}\dfrac{\left(2a\alpha_2+4\alpha_1\alpha_2\right)\left\{1-e^{\xi\alpha_1}\cos\xi\alpha_2\right\}\cos 2\alpha_1\alpha_2\tau-e^{\xi\alpha_1}\sin\xi\alpha_2\sin 2\alpha_1\alpha_2\tau}{\left(2\alpha_1^2-2\alpha_2^2-2a\alpha_1\right)^2+\left(2a\alpha_2+4\alpha_1\alpha_2\right)^2}+\\ \left(2\alpha_1^2-2\alpha_2^2-2a\alpha_1\right)\left\{\left(1-e^{\xi\alpha_1}\cos\xi\alpha_2\right)\sin 2\alpha_1\alpha_2\tau+e^{\xi\alpha_1}\sin\xi\alpha_2\cos 2\alpha_1\alpha_2\tau\right\}\end{matrix}\right]$$
$$-\frac{e^{\frac{s}{2}\xi-\left(\alpha_2^2-\alpha_1^2\right)\tau}}{\sqrt{\pi\tau}}\left[\begin{matrix}\dfrac{\left(2a\alpha_2+4\alpha_1\alpha_2\right)\left\{e^{-\xi\alpha_1}\cos\xi\alpha_2-1\right\}\cos 2\alpha_1\alpha_2\tau-e^{-\xi\alpha_1}\sin\xi\alpha_2\sin 2\alpha_1\alpha_2\tau}{\left(2\alpha_1^2-2\alpha_2^2-2a\alpha_1\right)^2+\left(2a\alpha_2+4\alpha_1\alpha_2\right)^2}\\ +\left(2\alpha_1^2-2\alpha_2^2-2a\alpha_1\right)\left[e^{-\xi\alpha_1}\sin\xi\alpha_2\cos 2\alpha_1\alpha_2\tau-\left(1-e^{-\xi\alpha_1}\cos\xi\alpha_2\right)\sin 2\alpha_1\alpha_2\tau\right]\end{matrix}\right], \quad (12)$$

Where

$$\alpha_1=\left[\frac{1}{2}\sqrt{\left(\frac{S^2}{2}+N+\lambda\right)^2+1}+\left(\frac{S^2}{2}+N+\lambda\right)\right]^{\frac{1}{2}},$$

$$\alpha_2=\left[\frac{1}{2}\sqrt{\left(\frac{S^2}{2}+N+\lambda\right)^2+1}-\left(\frac{S^2}{2}+N+\lambda\right)\right]^{\frac{1}{2}}.$$

For blowing $S<0$, say $S=-\tilde{S}$, the solutions (11) and (12)

$$\frac{f}{\Omega 1}=1+\frac{e^{\frac{\tilde{s}}{2}\xi+\xi\tilde{\alpha}_1}[2\tilde{\alpha}_1+2-\beta\tilde{S}]\cos\xi\tilde{\alpha}_2-2\tilde{\alpha}_2 2\sin\xi\tilde{\alpha}_2}{\left(2\tilde{\alpha}_1+2-\beta\tilde{S}\right)^2+(2\tilde{\alpha}_2)^2}-$$
$$\frac{e^{\left(-\frac{\tilde{s}}{2}\xi+\tilde{\alpha}_2^2-\tilde{\alpha}_1^2\right)\tau}}{\sqrt{\pi\tau}}\left\{\frac{\left(a^2+\tilde{\alpha}_2^2-\tilde{\alpha}_1^2\right)\left\{\cos 2\tilde{\alpha}_1\tilde{a}_2\tau+2\tilde{\alpha}_1\tilde{a}_2\sin 2\tilde{\alpha}_1\tilde{a}_2\tau\right\}\left(1-e^{\xi a}\right)}{\left(a^2+\tilde{\alpha}_2^2-\tilde{\alpha}_1^2\right)^2+\left(2\tilde{\alpha}_1\tilde{\alpha}_2\right)^2}\right\}$$
$$-\frac{e^{\left(-\frac{\tilde{s}}{2}\xi+\tilde{\alpha}_2^2-\tilde{\alpha}_1^2\right)\tau}}{\sqrt{\pi\tau}}\left\{\begin{matrix}\dfrac{\left(2\tilde{\alpha}_1^2-2\tilde{\alpha}_2^2-2a\tilde{\alpha}_1\right)\left\{\left(1-e^{\xi\tilde{\alpha}_1}\cos\xi\tilde{\alpha}_2\right)\cos 2\tilde{\alpha}_1\tilde{a}_2\tau-e^{\xi\tilde{\alpha}_1}\sin\xi\tilde{\alpha}_2\sin 2\tilde{\alpha}_1\tilde{a}_2\tau\right\}}{\left(2\tilde{\alpha}_1^2-2\tilde{\alpha}_2^2-2a\tilde{\alpha}_1\right)^2+\left(2a\tilde{\alpha}_2+4\tilde{\alpha}_1 a_2\right)^2}+\\ \left(2a\tilde{\alpha}_2+4\tilde{\alpha}_1\tilde{\alpha}_2\right)\left\{\left(1-e^{\xi\tilde{\alpha}_1}\cos\xi\tilde{\alpha}_2\right)\sin 2\tilde{\alpha}_1\tilde{a}_2\tau+e^{\xi\tilde{\alpha}_1}\sin\xi\tilde{\alpha}_2\cos 2\tilde{\alpha}_1\tilde{a}_2\tau\right\}\end{matrix}\right\}-$$
$$\frac{e^{\left(-\frac{\tilde{s}}{2}\xi+\tilde{\alpha}_2^2-\tilde{\alpha}_1^2\right)\tau}}{\sqrt{\pi\tau}}\left\{\begin{matrix}\dfrac{\left(2\tilde{\alpha}_1^2-2\tilde{\alpha}_2^2-2a\tilde{\alpha}_1\right)\left(\left(1-e^{-\xi\tilde{\alpha}_1}\cos\xi\tilde{\alpha}_2\right)\cos 2\tilde{\alpha}_1\tilde{\alpha}_2\tau+e^{-\xi\tilde{\alpha}_1}\sin\xi\tilde{\alpha}_2\sin 2\tilde{\alpha}_1\tilde{a}_2\tau\right)}{\left(2\tilde{\alpha}_1^2-2\tilde{\alpha}_2^2-2a\bar{\alpha}_1\right)^2+\left(2a\tilde{\alpha}_2+4\tilde{\alpha}_1\tilde{\alpha}_2\right)^2}+\\ \left(2a\tilde{\alpha}_2+4\tilde{\alpha}_1\tilde{\alpha}_2\right)\left(\left(e^{-\xi\tilde{\alpha}_1}\sin\xi\tilde{\alpha}_2\right)\cos 2\tilde{\alpha}_1\tilde{\alpha}_2\tau-(1-e^{-\xi\tilde{\alpha}_1}\cos\xi\tilde{\alpha}_2)\sin 2\tilde{\alpha}_1\tilde{a}_2\tau\right)\end{matrix}\right\}, \quad (13)$$

$$\frac{g}{\Omega 1}=-\frac{e^{\frac{\tilde{s}}{2}\xi-\xi\tilde{\alpha}_1}}{\left(2\tilde{\alpha}_1+2-\beta\tilde{S}\right)^2+\left(2\tilde{\alpha}_2\right)^2}\left[2\tilde{\alpha}_2\cos\xi\tilde{\alpha}_2+\left(2\tilde{\alpha}_1+2-\beta\tilde{S}\right)\sin\xi\tilde{\alpha}_2\right]$$
$$-\frac{e^{\frac{\tilde{s}}{2}\xi+\left(\tilde{\alpha}_2^2-\tilde{\alpha}_1^2\right)\tau}}{\sqrt{\pi\tau}}\left[\frac{2\tilde{\alpha}_1\tilde{\alpha}_2\cos 2\tilde{\alpha}_1\tilde{\alpha}_2\tau-\left(a^2+\tilde{\alpha}_2^2-\tilde{\alpha}_1^2\right)\left(1-e^{\xi a}\right)\sin 2\tilde{\alpha}_1\tilde{\alpha}_2\tau}{\left(-a^2-\tilde{\alpha}_2^2+\tilde{\alpha}_1^2\right)^2+\left(2\tilde{\alpha}_1\tilde{\alpha}_2\right)^2}\right]$$
$$-\frac{e^{\frac{\tilde{s}}{2}\xi-\left(\tilde{\alpha}_2^2-\tilde{\alpha}_1^2\right)\tau}}{\sqrt{\pi\tau}}\left[\begin{matrix}\dfrac{\left(2a\tilde{\alpha}_2+4\tilde{\alpha}_1\tilde{\alpha}_2\right)\left\{1-e^{\xi\tilde{\alpha}_1}\cos\xi\tilde{\alpha}_2\right\}\cos 2\tilde{\alpha}_1\tilde{\alpha}_2\tau-e^{\xi\tilde{\alpha}_1}\sin\xi\tilde{\alpha}_2\sin 2\tilde{\alpha}_1\tilde{\alpha}_2\tau}{\left(2\tilde{\alpha}_1^2-2\tilde{\alpha}_2^2-2a\tilde{\alpha}_1\right)^2+\left(2a\tilde{\alpha}_2+4\tilde{\alpha}_1\tilde{\alpha}_2\right)^2}+\\ \left(2\tilde{\alpha}_1^2-2\tilde{\alpha}_2^2-2a\tilde{\alpha}_1\right)\left\{\left(1-e^{\xi\tilde{\alpha}_1}\cos\xi\tilde{\alpha}_2\right)\sin 2\tilde{\alpha}_1\tilde{\alpha}_2\tau+e^{\xi\tilde{\alpha}_1}\sin\xi\tilde{\alpha}_2\cos 2\tilde{\alpha}_1\tilde{\alpha}_2\tau\right\}\end{matrix}\right]$$
$$-\frac{e^{\frac{\tilde{s}}{2}\xi-\left(\tilde{\alpha}_2^2-\tilde{\alpha}_1^2\right)\tau}}{\sqrt{\pi\tau}}\left[\begin{matrix}\dfrac{\left(2a\tilde{\alpha}_2+4\tilde{\alpha}_1\tilde{\alpha}_2\right)\left\{e^{-\xi\tilde{\alpha}_1}\cos\xi\tilde{\alpha}_2-1\right\}\cos 2\tilde{\alpha}_1\tilde{\alpha}_2\tau+e^{-\xi\tilde{\alpha}_1}\sin\xi\tilde{\alpha}_2\sin 2\tilde{\alpha}_1\tilde{\alpha}_2\tau}{\left(2\tilde{\alpha}_1^2-2\tilde{\alpha}_2^2-2a\tilde{\alpha}_1\right)^2+\left(2a\tilde{\alpha}_2+4\tilde{\alpha}_1\tilde{\alpha}_2\right)^2}\\ +\left(2\tilde{\alpha}_1^2-2\tilde{\alpha}_2^2-2a\tilde{\alpha}_1\right)\left[e^{-\xi\tilde{\alpha}_1}\sin\xi\tilde{\alpha}_2\cos 2\tilde{\alpha}_1\tilde{\alpha}_2\tau-\left(1-e^{-\xi\tilde{\alpha}_1}\cos\xi\tilde{\alpha}_2\right)\sin 2\tilde{\alpha}_1\tilde{\alpha}_2\tau\right]\end{matrix}\right] \quad (14)$$

With

$$\tilde{\alpha}_1=\left[\frac{1}{2}\sqrt{\left(\frac{\tilde{S}^2}{2}+N+\lambda\right)^2+1}+\left(\frac{\tilde{S}^2}{2}+N+\lambda\right)\right]^{\frac{1}{2}},$$

$$\tilde{\alpha}_2=\left[\frac{1}{2}\sqrt{\left(\frac{\tilde{S}^2}{2}+N+\lambda\right)^2+1}-\left(\frac{\tilde{S}^2}{2}+N+\lambda\right)\right]^{\frac{1}{2}}$$

RESULTS AND DISCUSSION

This area displays the graphical illustration of velocity profiles for the flow analyzed in this investigation .We interpret these results with respect to the variation of emerging parameters of interest and verify that they are consistent physically. Of particular interest here are the effects of the parameter λ, the slip parameter β and the suction/blowing parameter S.

In order to study the effects of various parameters of

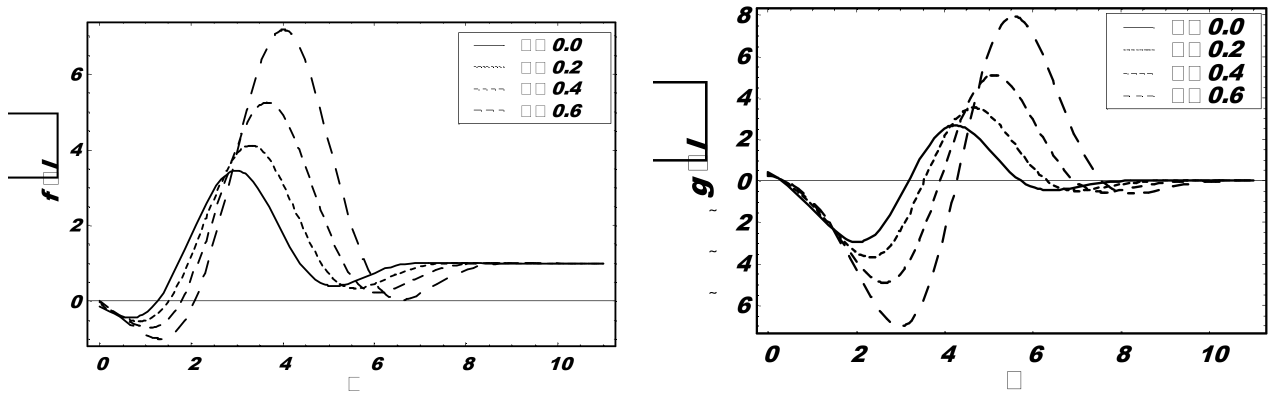

Figure 1. Variations of the velocity profiles for different values of λ when $S=0.2, N=0.1, \beta=0.1 \, and \, \tau=1.4$ are fixed.

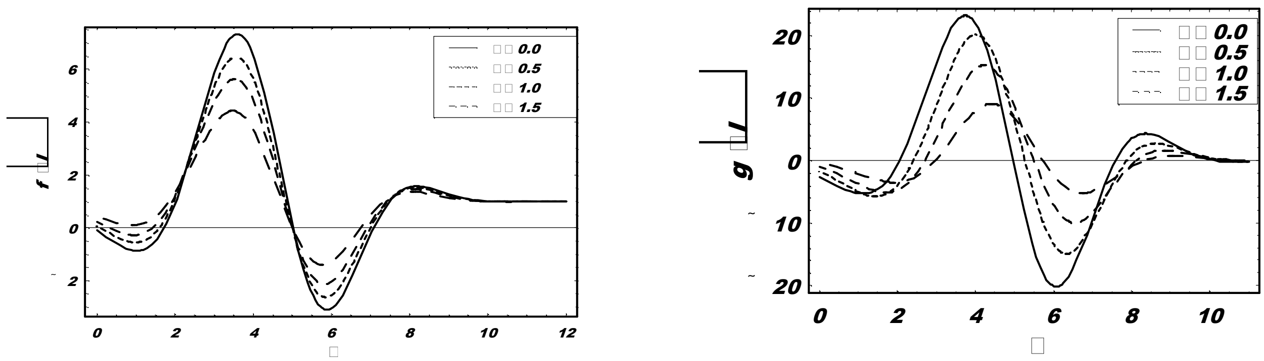

Figure 2. Variations of the velocity profiles for different values of β when $S=0.2, N=0.1, \beta=0.1 \, and \, \tau=1.4$ are fixed.

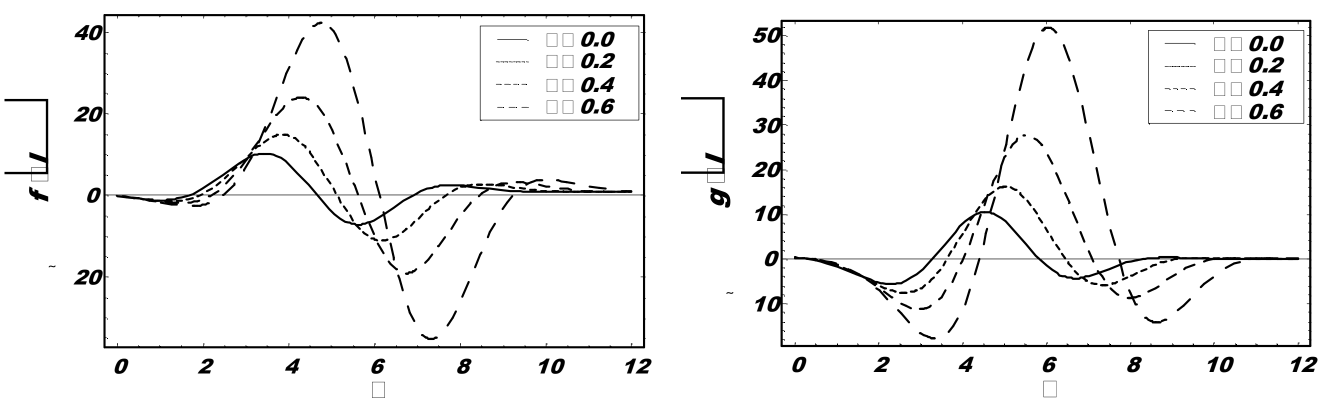

Figure 3. Variations of the velocity profiles for different values of λ when $S=-0.2, \; N=0.1, \; \beta=0.1 \; and \; \tau=1.4$ are fixed.

interest on the velocity distributions we have plotted $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ against ξ in Figures 1 to 4. Figures 1 and 2 are for the suction case while Figures 3 and 4 are for blowing.

Figures 1 and 3 shows the effects of λ on the velocity profiles. It is noted that with the increase of λ the velocity profiles decreases near the disk in both cases (suction/blowing) by keeping other parameters fixed. This is in accordance with the fact that, the increase of the porosity of the porous medium increases the drag force

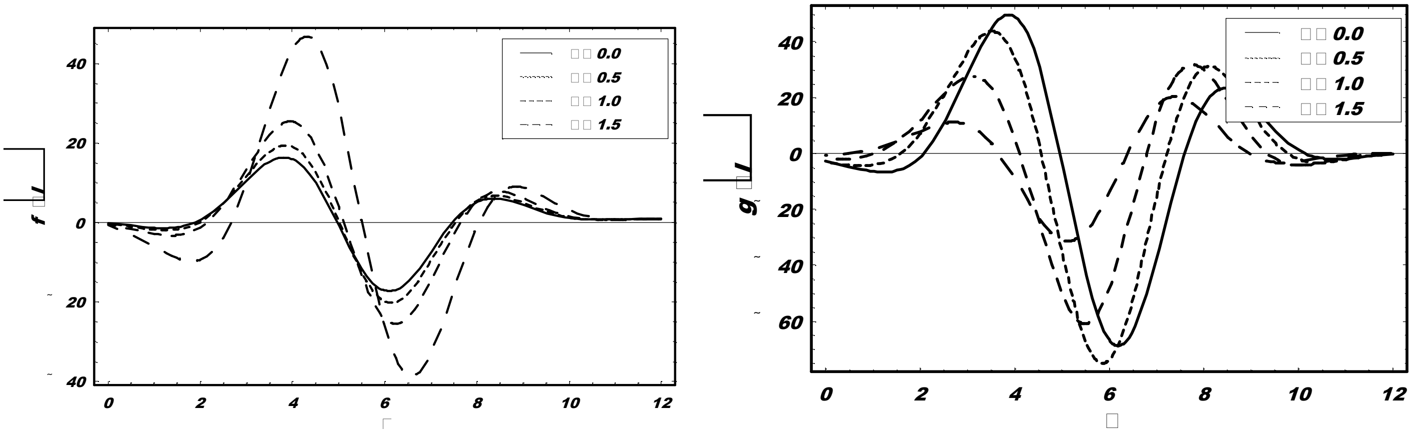

Figure 4. Variations of the velocity profiles for different values of β when $S=-0.2,\ N=0.1,\ \beta=0.1\ and\ \tau=1.4$ are fixed.

and hence causes the flow velocity to decrease. Thus, increasing the parameter λ yields an effect opposite to that of the permeability.

Figures 2 and 4 are prepared to elucidate the influence of the slip parameter β on the velocity profiles with fixed values of the other parameters. It is found that with the increase of slip parameter β, the velocity gradient decreases and hence the velocity profiles become flatter due to the decreasing shearing force from the slip boundary. It is also noted that the results of Hayat et al. (2001) can be recovered by taking $\beta=0=\lambda$.

CONCLUDING REMARKS

This article describes the unsteady flow of an incompressible electrically conducting fluid with partial slip characteristics in a porous medium. This is a theoretical study and may have applications in engineering. Mathematical analysis is performed in the presence of slip effects. The main points can be extracted from the present investigation.

1) It is found that with the increase of slip parameter β, the velocity gradient decreases.

2) The effects of λ yields effect opposite to that of the permeability k.

3) It is also noted that the results of Hayat et al. (2001) can be recovered by taking $\beta=0=\lambda$

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