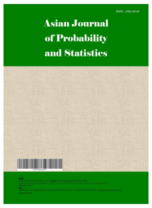

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Radial Basis Function in Nonlinear Black-Scholes Option Pricing Equation with Transaction Cost

Godwin Onwona-Agyeman[1*] and Francis T. Oduro[1,2]

[1]*Department of Mathematics, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana.*
[2]*African Institute of Mathematical Sciences, Accra, Ghana.*

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final Manuscript.

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Abstract

Differential equations play significant role in the world of finance since most problems in these areas are modeled by differential equations. Majority of these problems are sometimes nonlinear and are normally solved by the use of numerical methods. This work takes a critical look at Nonlinear Black-Scholes model with special reference to the model by Guy Barles and Halil Mete Soner. The resulting model is a nonlinear Black-Scholes equation in which the variable volatility is a function of the second derivative of the option price. The nonlinear equation is solved by a special class of numerical technique, called, the meshfree approximation using radial basis function. The numerical results are presented in diagrams and tables.

Keywords: Black-Scholes; radial basis function; differential equations.

**Corresponding author: E-mail: juniorzee@yahoo.com;*

1 Introduction

A financial derivative is a financial instrument whose value is contingent on the value of the underlying asset. Forwards, futures and options are some of the simplest financial derivatives we have. An option can be described as a contract or financial agreement which gives its owner the right to buy or sell an underlying asset at a fixed price, say, K, at any time on or prior to a given date, T. There two main types options, namely, the call option which gives its owner the right to purchase or buy an option and a put option which gives its owner the right to sell or trade the option. The value of an option $V(s,t)$ is dependent on the price of the underlying asset S and the time t. The standard Black-Scholes partial differential equation is generally used to value the European option. The Black-Scholes partial differential equation was developed by Fischer Black and Myron Scholes in 1973 [1].

During [2] discussed that some of the assumptions used in the formulation of the Black-Scholes model proves to be too restrictive in practice. For example, some of the assumptions stated were that, interest rate is known and constant through time and transactions cost do not exist in hedging a portfolio whereas in the real world, interest rate is not fixed or constant and transaction cost do exist. Since then, there have been several and fruitful reformulation of Black-Scholes model that take into account interest rate or transaction costs. The resulting equation of the Black-Scholes model becomes nonlinear for the option price. In this case, to compute the solution by the analytical formula of the standard Black-Scholes equation is difficult and hence has to resort to numerical methods or approximations.

In this paper, we study the Black-Scholes equation with transaction cost with a special reference to the model by Guy Barles and Halil Mete Soner. We then apply the Meshfree radial basis function (RBF) as a spatial approximation for the solution of the option value.

The meshfree methods have recently been studied by a couple of authors. Kansa [3] used radial basis function (RBF) to solve some of the problems in computational fluid dynamics. Kansa used the multi-quadric radial basis function for parabolic, hyperbolic and elliptic PDEs.

Sharan et al. [4] applied the multiquadric RBF to obtain the solution of elliptic PDEs with Dirichlet and Neumann boundary conditions. He concluded that, the results found by the RBF and the exact/analytical solutions were very good.

Goto et al. [5] studied options valuation by using radial basis function approximation whiles Belova et al. (2008) presented meshfree approximation based on RBF for the numerical solution of European, Barrier, Asian and American options.

Milovanovic and Shcherbakov [6] proposed two localized RBF methods, for solving financial derivative pricing problems arising from models with multiple stochastic factors.

2 Materials and Methods

2.1 Nonlinear Black-Scholes equation

The Black-Scholes model, given by equation (2.1) below

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV \tag{2.1}$$

was derived by Fischer Black and Myron Scholes in 1973 based on certain assumptions, such as "no transaction cost." Thus, when transaction cost is incorporated into the standard Black-Scholes equation, the model result in fully nonlinear Black-Scholes model according to during [2] and stated the nonlinear equation as:

$$V_t + \frac{1}{2}\hat{\sigma}(S,t,V_s,V_{ss})^2 S^2 V_{ss} + rSV_s - rV = f(S,t,V_s), \quad S > 0, \ \ t \in (0,T) \tag{2.2}$$

Where r denotes the riskless interest rate, $\hat{\sigma}$ is the nonlinear volatility, and the nonlinear function f models different effects.

2.2 Volatility models

In the Black-Scholes model, the parameter assumed to be constant is the volatility σ. Ankudinova [7] stated that there are many different approaches which have been developed to improve the model by treating the volatility in different ways using modified volatility function, $\hat{\sigma}(.)$ to modify the effects of transaction cost. Some of the modified volatility models developed are the model by Leland [8], Model of Hodges and Neuberger [9], Boyle and Vorst developed in 1992 [10], and the model by Barles and Soner developed in 1998 [11] .

2.2.1 Transaction cost model of Barles and Soner

Barles & Soner [11] apply the utility function approach of Hodges & Neuberger [9] with the help of an asymptotic analysis of partial differential equations. Considering a financial market which consists of a single bond and a single stock, the price is characterized by:

$$dS(l) = S(l)[\alpha dl + \sigma dW(l)], \quad l \in [t, T] \tag{2.3}$$

with initial data $S(t) = s$, $W(.)$ is a unit-dimensional Brownian motion, α is the constant mean return rate and σ is the constant volatility.

According to Barles and Soner, if the process of bonds owned $X(s)$ and the process of shares owned $Y(s)$, then the trading strategy $(L(s), M(s))$ can be a pair of non-decreasing processes with $L(t) = M(t) = 0$, which are interpreted as the cumulative transfers measured in shares of stock. $M(s)$ is measured in shares from stock to bond and $L(s)$ is measured in shares from bond to stock. Let $\mu \in (0,1)$ be the proportional transaction cost and initial values x and y, $s \in [t, T]$,

The processes X (s) and Y (s) evolve according to

$$X(s) = x - \int_t^s S(\tau)(1+\mu)dL(\tau) + \int_t^s S(\tau)(1-\mu)dM(\tau), \quad s \in [t,T] \tag{2.4}$$

and

$$Y(s) = y + L(s) - M(s), \quad s \in [t,T] \tag{2.5}$$

where the first integral represents buying shares of stock at a price increased by the proportional transaction cost μ and the second integral represents selling stock at a reduced price of the transaction cost. We add the amount of the stocks bought and subtract the amount for the stocks sold to the initial amount of stocks owned in equation (2.5). According to the utility maximization approach of Hodges & Neuberger, the price of a European Call option can be found by the difference between the maximum utility of the terminal wealth when there is no option liability and when there is such a liability. Let the exponential utility function be

$$U(\xi) = 1 - e^{\gamma\xi}, \ \ \xi \in \mathbb{R} \tag{2.6}$$

where $\gamma > 0$ is the risk aversion factor. Boyle & Vorst [10] considered two optimization problems. The first value function is the expected utility from the final wealth when there are no option liabilities taken over the transfer processes

$$V_a(x, y, S(t), t) := \sup_{L(.)M(.)} E\left[U\left(X(T) + Y(T)S(T)\right)\right] \tag{2.7}$$

and the second value function is the expected utility from the final wealth assuming that we have sold N European call options taken over the transfer processes

$$V_b(x, y, S(t), t) := \sup_{L(.)M(.)} E\left[U\left(X(T) + Y(T)S(T) - N((T) - K)^+\right)\right] \tag{2.8}$$

According to Hodges & Neuberger [9] the price of each option is equal to the maximal solution Λ of the algebraic equation

$$V_a(x + N\Lambda, y, S(t), t) = V_b(x, y, S(t), t)$$

This means that the option price L equals the increment of the initial capital at time t that is needed to cope with the option liabilities arising at T Ankudinova [7]. By a linearity argument selling N options with risk aversion factor of γ yields the same price as selling one option with risk aversion factor γN.

This leads to performing an asymptotic analysis as $\gamma N \longrightarrow inf$. We then consider

$$U(\xi) = 1 - e^{-\gamma N \xi}$$

We set

$$\gamma N = 1/\varepsilon$$

Then, we have

$$U_\varepsilon(\xi) = 1 - e^{-\xi/\varepsilon} \qquad \xi \in \mathbb{R}$$

Our optimization problem then becomes

$$V_a(x, y, S(t), t) = 1 - \inf_{L(.)M(.)} E\left[e^{-1/\varepsilon(X(T) + Y(T)S(T))}\right]$$

and

$$V_b(x, y, S(t), t) = 1 - \inf_{L(.)M(.)} E\left[e^{-1/\varepsilon(X(T) + Y(T)S(T) - (S(T) - K)^+}\right]$$

Now define, z_a and $z_b : \mathbb{R} \times (0, \infty) \times (0, T) \longrightarrow \mathbb{R}$ by

$$V_a(x, y, S(t), t) = 1 - e^{-1/\varepsilon(x + yS(t) - z_a(y, S(t), t))}$$
$$V_b(x, y, S(t), t) = 1 - e^{-1/\varepsilon(x + yS(t) - z_b(y, S(t), t))}$$

Then

$$z_a(y, S(t), T) = 0 \text{ and } z_b(y, S(t), T) = (S(T) - K)^+$$

And the option price is given by

$$\Lambda\left(x, y, S(t), t, \frac{1}{\varepsilon}, 1\right) = z_b(y\,, S(t\,), t) - z_a(y\,, S(t\,), t)$$

Barles & Soner [11] state that the value functions V_a and V_b are the unique solutions of the dynamic programming equation

$$min\left\{-V_t + \frac{1}{2}\sigma^2 S^2 V_{ss} - rSV_s, -V_y + S(1+\mu)V_x, -V_y + S(1-\mu)V_x\right\} = 0 \tag{2.9}$$

by the theory of stochastic optimal control. This leads to a dynamic programming equation for z_a and z_b, which are independent of the variable x. Supposing that the proportional transaction cost $\mu = a\sqrt{\varepsilon}$ for some constant $a > 0$, Barles & Soner prove that as $\varepsilon \longrightarrow 0$ and $\mu \longrightarrow 0$, $z_a \longrightarrow 0$ and $z_a \longrightarrow V$,

$$V_t + \frac{1}{2}\hat{\sigma}^2 S^2 V^2{}_{ss} + rSV_s - rV = 0 \tag{2.10}$$

where

$$\hat{\sigma}^2 = \sigma^2\big(1 + \Psi(e^{r(T-t)}a^2 S^2 V_{ss}\big) \tag{2.11}$$

is the nonlinear volatility. σ denotes the constant volatility of the underlying, $a = \mu/\sqrt{\varepsilon}$ and $\Psi(x)$ is the solution to the following nonlinear ordinary differential equation

$$\Psi'(x) = \frac{\Psi(x)+1}{2\sqrt{x\Psi(x)-x}} \qquad x \neq 0 \tag{2.12}$$

with the initial condition

$$\Psi(0) = 0 \tag{2.13}$$

Barles & Soner analyzed the ordinary differential equation in equations (2.12) and (2.13) and stated that

$$\lim_{n\to\infty}\frac{\Psi(x)}{x} = 1 \quad \text{and} \quad \lim_{n\to-\infty}\frac{\Psi(x)}{x} = -1 \tag{2.14}$$

Because of equation (2.14), the function $\Psi(.)$ is treated as identity for large arguments. The volatility then becomes

$$\hat{\sigma}^2 = \sigma^2\big(1 + (e^{r(T-t)}a^2 S^2 V_{ss}\big) \tag{2.15}$$

2.3 Meshfree approximation

A radial Basis function is a real valued function whose value depends only on the distance from some other point, c, called center such that

$$\phi(x, c) = \phi(\parallel x - c \parallel).$$

According to Fasshauer [12], the Radial Basis Function (RBF) approximation deals with a single variable basis function and a specific Euclidean norm that decreases a multi-dimensional problem into a unit-dimensional problem. Duffy [13] stated that the radial basis function method does not depend on the dimension of the problem. Fasshauer explained that, the radial basis function method approximates the value of a function as the weighted sum of radial basis functions. These functions are then evaluated on a set of points called centers, which are quasi-randomly scattered over the domain of the problem, Guarin et al. [14]. The weights are found by matching the approximated and observed values of the function. Once the weights are calculated, they are used to approximate the value of the function at any point over the entire domain.

Following Fasshauer [12], we consider the set of nodes or centers $X = [x_1, x_2, \cdots, x_N]'$ with $x_j \in \mathbb{R}^s, s \geq 1$ and the data values $g_j \in \mathbb{R}$. We assumed that $g_j = f(x_j, t)$ $j = 1,2,3, \cdots, N$ where f is an unknown function and t is the time. Let $f(X, t)$ be a linear combination of N certain basis functions such that

$$f(X,t) \simeq \sum_{j=1}^{N} b_j(t)\phi(\| x - c \|) \tag{2.16}$$

where the coefficients $b_j(t)$ are the unknown weights, $\phi(\cdot)$ is the radial basis function and $\|\cdot\|$ Euclidean norm. Equation 2. 16 can be represented as a system of linear equations.

2.4 Application of radial basis function

An efficient numerical method is developed based on meshfree approximation using radial basis function which would be used to solve the nonlinear Black-Scholes model using the transaction cost model of Guy Barles and Halil Mete Soner. This model leads to a system of differential equations which is solved by a time integration scheme. This is achieved by using the theta (θ) method.

2.4.1 Discretization

We approximate the value of the option, V; in the nonlinear Black-Scholes equation using the radial basis function as

$$V(S,t) \simeq \sum_{j=1}^{N} b_j(t)\phi(S, S_j) \tag{2.17}$$

where b_j are unknown coefficients and $\phi(S, S_j)$ are the radial basis functions. Multiquadric radial basis function (MQ-RBF) would be used for this problem

$$\phi(S, S_j) = \sqrt{c^2 + \| S - S_j \|^2} \tag{2.18}$$

Where S_j is the asset price at the collocation point j for approximating the option price V, $\| S - S_j \|$ denotes the radial distance of each of the N scattered data points S_j. The parameter c is positive and known as shape parameter.

It is well known that the following nonlinear Black-Scholes equation holds for the option price $V(S,t)$ with asset price S at time t

$$\frac{\partial V(S,t)}{\partial t} + \frac{1}{2}\hat{\sigma}^2 S^2 \frac{\partial^2 V(S,t)}{\partial S^2} + rS\frac{\partial V(S,t)}{\partial S} - rV = 0 \tag{2.19}$$

With the nonlinear term given as

$$\hat{\sigma}^2 = \sigma^2\big(1 + (e^{r(T-t)}a^2 S^2 V_{ss}\big) \tag{2.20}$$

and the initial condition given by the terminal payoff function

$$V(S,t) = \begin{cases} max\ S(T\ )\quad K;\ 0\ for\ call\ option \\ max\ K - S(T\ ), 0\ for\ put\ option \end{cases}$$

where T is the time of maturity and K is the strike price.

Differentiating equation 2.17, we obtain the following

$$\frac{\partial V(S,t)}{\partial t} = \sum_{j=1}^{N} \frac{db_j(t)}{dt} \phi(S, S_j) \tag{2.21}$$

$$\frac{\partial V(S,t)}{\partial S} = \sum_{j=1}^{N} b_j \frac{\partial \phi(S,S_j)}{\partial S} \tag{2.22}$$

$$\frac{\partial^2 V(S,t)}{\partial S^2} = \sum_{j=1}^{N} b_j \frac{\partial^2 \phi(S,S_j)}{\partial S^2} \tag{2.23}$$

and the derivatives of equation 2.18 are also given by

$$\frac{\partial \phi}{\partial S} = \frac{S-S_j}{\sqrt{(S-S_j)^2+c^2}} \tag{2.24}$$

$$\frac{\partial^2 \phi}{\partial S^2} = \frac{1}{\sqrt{(S-S_j)^2+c^2}} - \frac{(S-S_j)^2}{\sqrt{((S-S_j)^2+c^2)^3}} \tag{2.25}$$

Following the approach in the article of Gonzalez-Gaxiola & Gonzalez-Perez [15], We obtain

$$HV(S, t + \Delta t) = GV(S, t) \tag{2.26}$$

by the application of the theta method of the Crank-Nicholson scheme, where

$$\begin{aligned} G &= 1 - \theta\Delta t \left(\frac{1}{2}S^2 \frac{\partial^2}{\partial S^2}\left(1 + \left[e^{r(T-t)}a^2 S^2 \frac{\partial^2}{\partial S^2}\right]\right) + rS\frac{\partial}{\partial S} - r\right) \\ H &= 1 + (1-\theta)\Delta t \left(\frac{1}{2}S^2 \frac{\partial^2}{\partial S^2}\left(1 + \left[e^{r(T-t)}a^2 S^2 \frac{\partial^2}{\partial S^2}\right]\right) + rS\frac{\partial}{\partial S} - r\right) \end{aligned}$$

The option price which is governed by equation 2.19 would be estimated with the radial basis function, equation 2.17. Substituting equation 2.17 into equation 2.26 leads to

$$Hb_j(t + \Delta t)\phi \parallel S - S_j \parallel = Gb_j(t)\phi \parallel S - S_j \parallel \tag{2.27}$$

Equation 2.27 is solved for b_j. Thus equation 2.27 is solved iteratively from the expiration date $t = T$ to the date of purchase $t = 0$. Once the parameter $b(0)$ is obtained at the date $t = 0$, the option price at the date of purchase is estimated from $b(0)$ by using

$$V(S,t) \simeq \sum_{j=1}^{N} b_j(t)\phi(S,S_j)$$

3 Numerical Results

Using the multi quadric radial basis (equation 2.18) function approach, the resulting problems for the Black-Scholes model with transaction cost is solved via Crank Nicolson's method. The parameters are presented in Table 1.

The parameter values are provided by Belova and Shmidt, [16] with the exception of the shape parameter a from Mawah [17]. We consider an algorithm that was suggested in Goto et al [5] for evaluating the option price,$V(S,t)$. The reader can see the algorithm in Goto et al [5] and Belova et al [16 for more details.

All the values and diagrams obtained for the European option with and without transaction cost were implemented in MATLAB R2018b.

Table 1. Parameter values

Parameter	Description	Value
S_{max}	Maximum asset price	30
N	Number of asset data points	121
Δt	Number of time steps	0.05
T	Expiration date	0.5
K	Exercise price	10
r	Risk free interest rate	0.05
σ	Volatility	0.2
θ	Crank-Nicholson method	0.5
c	Shape parameter	0.01
a	Transaction Cost parameter	0.005

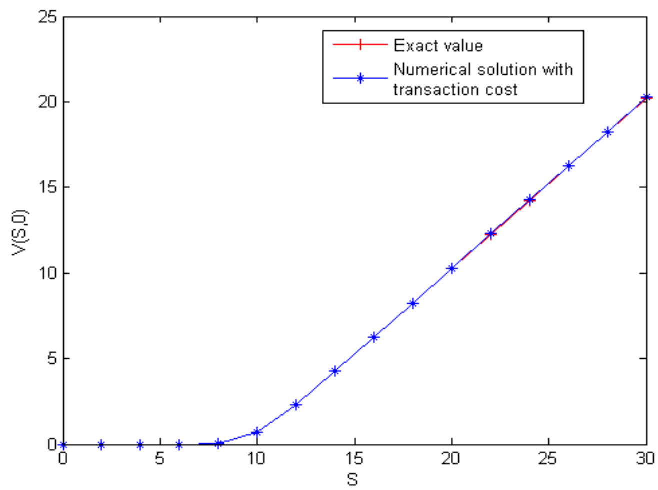

Fig. 1. Values of the nonlinear option price in comparison with the exact solution

Table 2. The comparison of exact value and option value with transaction coast

Asset price, *S*	Exact value	Option value with transaction cost
0.0	0.0000	0.0000
2.0	0.0000	0.0000
4.0	0.0000	0.0000
6.0	0.0000	0.0000
8.0	0.0456	0.0561
10.0	0.6888	0.6983
12.0	2.2952	2.3104
14.0	4.2496	4.2681
16.0	6.2470	6.2781
18.0	8.2469	8.2500
20.0	10.2469	10.2960
22.0	12.2469	12.2877
24.0	14.2469	14.3010
26.0	16.2469	16.2899
28.0	18.2469	18.2654
30.0	20.2469	20.2743

Fig. 1 shows a comparison of results between the options values with transaction cost and the exact values or the analytical solution of the standard Black Scholes equation (equation 2.1). Even though the results seems identical, there is a substantial difference in the results as shown in Table 2. From Table 2, the exact value of the option and the option value with transaction cost are 20.2469 and 20.2743 respectively for a maximum asset price of 30.0. This means that, the numerical scheme used approximates the option value produces a minimum error of less than 5% as the asset price increases.

Fig. 2 depicts the superiority of options with transaction cost over options without transaction cost as the graph without cost diverges slightly away for higher values of the asset price, S.

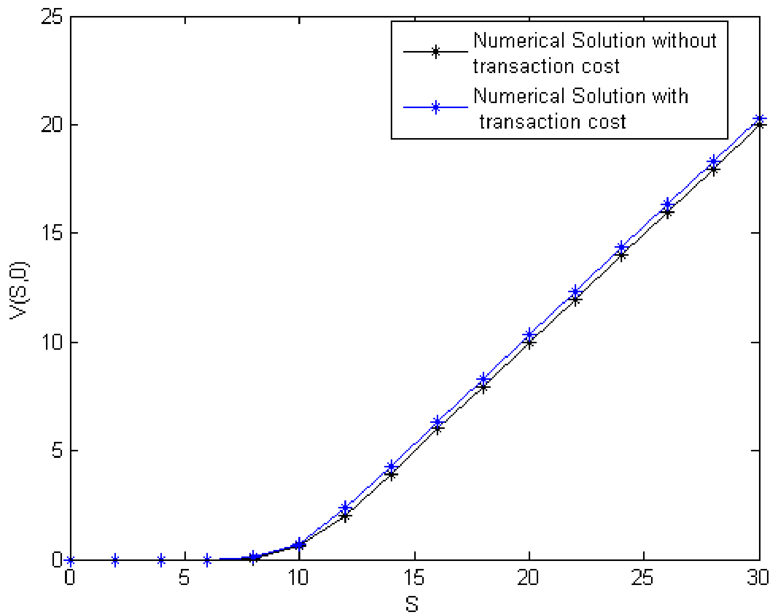

Fig. 2. The option values with and without transaction cost

Table 3. Option values with transaction cost

Asset Price, S	**Option Price with $a = 0.01$**	**Option Value with $a = 0.005$**
0.0	0.0000	0.0000
2.0	0.0000	0.0000
4.0	0.0000	0.0000
6.0	0.0000	0.0000
8.0	0.0745	0.0561
10.0	0.7179	0.6983
12.0	2.3461	2.3104
14.0	4.2895	4.2681
16.0	6.3263	6.2781
18.0	8.3041	8.2500
20.0	10.3254	10.2960
22.0	12.3085	12.2877
24.0	14.3432	14.3010
26.0	16.3204	16.2899
28.0	18.2871	18.2654
30.0	20.3056	20.2743

The curves in Fig. 3 look identical as the difference between the option values with a = 0.005 and a = 0.01 is not much. For maximum asset price of 30, the option values with transaction costs a = 0.005 and a = 0.01 are 20.2743 and 20.3056 respectively. This can be confirmed from Table 3. It can clearly be seen that when the higher the value of a; the higher the value of the option price.

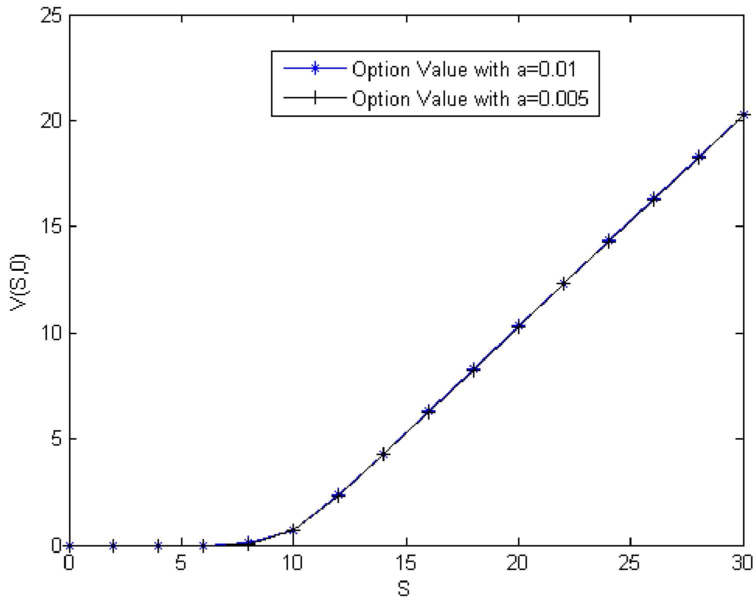

Fig. 3. Option values with transaction cost when a is 0.01 and 0.005

4 Conclusion

In this paper, we first reviewed option pricing and meshfree methods in general. We used a special class of numerical methods, namely, Meshfree Methods using radial basis function as a spatial approximation, to study the differential models for pricing options. The method was applied to solve the Black-Scholes standard equation in the presence transaction cost. The application of radial basis functions led to a system of differential equations which were solved by the Crank- Nicholson Scheme. The numerical results describing the payoff function and the values of the option were presented. We compared the influence of transaction costs on the value of the option to that of the standard Black-Scholes option price as well as the option value without transaction cost. The option values obtained in the presence of transaction costs were slightly higher in all cases.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Black F, Scholes, M. The pricing of options and corporate liabilities. The Journal of Political Economy. 1973;81:637-654.

[2] During, B. Black-Scholes type equations: Mathematical analysis, parameter identification & numerical solution. Unpublished doctoral dissertation, am Fachbereich Physik, Mathematik und Informatik der Johannes Gutenberg-Universitat Mainz; 2005.

[3] Kansa E. Multiquadrics- A scattered data approximation scheme with applications to computational fluid-dynamics. Computers & Mathematics with Applications. 1990;19:127-145.

[4] Sharan M, Kansa E, Gupta S. Application of the multiquadric method for numerical solution of elliptic partial differential equations. Applied Mathematics and Computation. 1997;84:275-302.

[5] Goto Y, Fei Z, Kan S, Kita E. Option valuation by using radial basis function. Engineering Analysis with Boundary Elements. 2007;31:836-843.

[6] Milovanovic S, Shcherbakov V. Pricing Derivatives under multiple stochastic factors by localized radial basis function methods, Uppsala University, Sweden; 2018.

[7] Ankudinova J. The numerical solution of nonlinear Black-Scholes equations. Unpublished doctoral dissertation, Technische Universitat Berlin; 2008.

[8] Leland HE. Option pricing and replication with transaction costs. The Journal of Finance. 1985;40: 1283-1301.

[9] Hodges S, Neuberger A. Optimal replication of contingent claims under transaction costs. The Review of Futures Markets. 1989; 8, 222-239.

[10] Boyle, P. P., & Vorst, T. Option replication in discrete time with transaction costs. The Journal of Finance. 1992;47(1):271-293.

[11] Barles G, Soner HM. Option pricing with transaction costs and a nonlinear Black-Scholes equation. Finance and Stochastics. 1998;2:369-397.

[12] Fasshauer GE. Meshfree methods. Department of Applied Mathematics, Illinois Institute of Technology IL 60616, USA; 2006.

[13] Duffy DJ. Finite difference methods in financial engineering a partial differential equation approach. John Wiley & Sons Ltd; 2006.

[14] Guarin A, Liu X, Ng WL. Valuation of american options with meshfree methods. (Centre for Computational Finance and Economic Agents, Working Paper Series; 2012.

[15] Gonzalez-Gaxiola O, Gonzalez-Perez PP. Nonlinear Black-Scholes equation through radial basis function. Journal of Applied Mathematics and Bioinformatics. 2014;4(3):75-86.

[16] Belova A, Shmidt T. Meshfree methods in option pricing. Unpublished master's thesis, School of Information Science, Computer and Electrical Engineering Halmstad University, School of Information Science, Computer and Electrical Engineering Halmstad University; 2011. (In press).

[17] Mawah B. Option pricing with transaction costs and a non-linear Black-Scholes equation. Unpublished Master's Thesis, Department of Mathematics, Uppsala University; 2007.

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