Transverse Motions of Bernoulli-euler Beam Resting on Elastic Foundation and under Two Concentrated Moving Loads

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

Original Research Article

Abstract

Aims/Objectives: The aim is to obtain a closed form solutions of single-dimensional structural element of continuously supported by an elastic foundation. Thereafter, we classify the effects of the space d connecting the loads on the relevant partial differential equations governing the motion of the structural members. The study also analysis circumstances under which resonance occur in the dynamical systems involving structural members.
Study Design: The single-dimensional structural element is a partial differential equation of order fourth order place on elastic Winkler foundation. The Bernoulli-Euler beam traversed by two moving loads.
Place and Duration of Study: Department of Mathematical Sciences, Adekunle Ajasin University P.M.B. 01, Akungba-Akoko, Nigeria, between May 2019 and September 2019.
Methodology: The principal equation of the single -dimensional beam model is governing by partial differential equation of the order four. For the single -dimensional beam problem, the solution techniques are based on the Fourier sine transformation. The governing partial differential equation of the order four was reduced to sequence of second order ordinary differential equations.

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Nomenclatures

$M[kg]$: Mass of the moving load

$L\ [m]$: Length of the beam

$S(x,t)\ [N]$: Shear force

$E\ [N/m^2]$: Modulus of elasticity

$v(x,t)\ [m]$: Deflection from the equilibrium

$P_i(x,t)\ [N]$: Moving concentrated forces

$\mu\ [kg/m]$: Mass of the beam per unit length L

$E_0\ [N/m^3]$: Elastic foundation modulus

$N(x)[N/m^3]$: Non-uniform prestressed

$d\ [m]$: Space or gap between the loads

$I\ [m^4]$: Moment of inertia of the beam,

$v\ [m/s]$: Constant speed

$x\ [m]$: The spatial and time coordinates

$t\ [s]$: Time coordinates

$\Omega\ [Hz]$: Circular frequency of the harmonic force

1 Introduction

The dynamic analysis of structural elements has received very much attention because of its relevant in various engineering applications. The applications include the response of railroad rails to moving trains, the response of bridges and elevated roadways to moving vehicles, machine chain and belt drives, computer tape drives, floppy disks and video cassette recorders. One of the early experimenters in the field is Kenny [1]. He investigated the dynamic response of infinite beam on elastic foundations under the action of moving load of constant speed. He included in the governing equation the effects of viscous damping. The limiting cases of no damping and critical damping were investigated. The Winkler foundation model consisting of an infinite number of closely, spaced springs uniformly distributed along the structure were considered. Another notable researcher in this field is Stanisic. The two dimensional problems of flexural vibrations of plates under the action of moving masses was investigated by Stanisic et al. [2]. Only the term that measures the effects of rate of velocity in the path of the deflection was considered. A method based on the Fourier transform technique was used and only simply supported boundary conditions were considered. The resulting coupled second order differential equation was solved via a numerical technique. The reaction of a variable cross section slight beam resting on a regular elastic foundation to several moving masses was considered by Oni [3]. For the solution of the problem, he used the versatile technique of Galerkin to reduce the complex governing partial differential equation of order four with variable and singular coefficients to a group of ordinary differential equations. The group of ordinary differential equations was later simplified and solved using modified asymptotic method of Struble. Ozkaya [4] studied Non-linear transverse motion of a beam with simply supported carrying concentrated masses. Dasa et al. [5] worked on the free out-of-plane vibration of a rotating beam with non-linear spring mass system using perturbation method and obtained the non-linear natural frequencies for a vibrating blade model. Lin et al. [6] investigated analysis of free vibration of a regular cross section multi-span transporting multiple spring-mass systems using Euler–Bernoulli beam theory. Oni and Adedowole [7] considered influence of prestressed on the response to

moving loads of rectangular plates incorporating rotatory inertia correction factor. In the study, versatile technique of Galerkin was used to reduce the complex governing partial differential equation of order four with variable and singular coefficients to a set of ordinary differential equations.

Yesilce and Catal [8] estimated normalized usual frequencies of the pile using carry-over matrix and incorporating rotatory inertia. Hosing et al. [9] carried out investigation on the solution to natural flexural excitation of a continuous structural element on discrete elastic support. Yusuf et al. [10] investigated free excitations of a several-span Timoshenko beam transporting various spring-mass systems. Frequency values and mode shapes for free excitation of the several-span Timoshenko single-dimensional with numerous spring-mass systems are obtained for diverse number of spans and spring-masses with diverse points in the work. Oni and Ogunyebi [11] considered the dynamic behaviour of specific number of prestressed Bernoulli-Euler beams with general boundary conditions to mobile evenly distributed weights. Their work incorporated the influence of rotatory inertia factor, prestressed (axial force) of the travelling equally distributed loads and as well as the effects of foundation moduli in the governing differential equation of the dynamical problem.

Recently, Jimoh [12] studied the motion of non-uniformly prestressed tapered beams with exponentially varying thickness resting on Vlasov foundation subjected to travelling variable harmonic load with steady velocity. Jafarzadeh et al. [13] investigated transverse vibration of nano-Timoshenko beams carrying multiple concentrated masses. They used Dirac's delta function to impose the mathematical model of the concentrated masses into the equations of motion, and presented the exact closed-form solution for this problem. Bakhshi [14] studied vibration behavior of rotating nanobeams. He used the local and nonlocal theories to model nanobeams, and reported the effects of diverse material and scale parameters on the mechanical behavior of them. According to the introduced literature, all the above-mentioned studies applied the Euler–Bernoulli beam theory, and presented without consideration of the effects of shear forces and rotating inertia in beams.

More recently, Malesela and Sarp [15] investigated small scale effects on the fundamental frequency for a nanobeam with elastically restrained end conditions and moving a tip mass attached by the use of a linear spring to the end of the beam. Transverse Response of a Structural Member with Time Dependent Boundary Conditions to Moving Distributed Mass was investigated by Omolofe and Adedowole [16]. In their work, closed form solutions of the governing fourth order partial differential equations with variable and singular coefficients of a beam-mass problem were presented. Jimoh and Ajoge [17] studied the behavior of regular Rayleigh beam resting on Pasternak foundation and traversed by exponentially varying magnitude moving load. Alireza et al. [18] solved the general term of mode shape function with four undetermined coefficients of the non-uniform beam with mass per unit length and bending moment of inertia varying as polynomial functions by via the generalized power series method.

Nevertheless, the case where we have two concentrated loads with constant and harmonic load is scanty in literatures.

Thus, this study sets to investigate response of structural elements resting on elastic foundation subjected to two concentrated moving loads.

2 Formulation of the Problem

Consider a Bernoulli-Euler beam having length L and laying on an elastic foundation traverse by travelling weight with steady velocity governed by the partial differential equation

$$\mu(x)v_{tt}(x,t) + \varepsilon_o v_t(x,t) + E_o v(x,t)$$

$$= \partial S_x(x,t) + N_0(x)v_{xx}(x,t) + P_i(x,t) \tag{2.0}$$

$$S_x(x,t) = -EI(x)v_{xx}(x,t) \qquad (2.1)$$

Where $S(x,t)$ is the shear force, $P_i(x,t)$ are the steady mobile weights acting on the structural material, μ is the mass of the beam per unit length L, $v(x,t)$ is the vertical deflection of the beam, $EI(x)$ is the flexural rigidity of the beam, E is the young modulus $N(x)$ is the non-uniform prestress, x and t are the spatial and time coordinates respective. The structure under consideration is simply supported and carrying a concentrated mass M, which is moving at constant velocity.

3 The Simply Supports Boundary Conditions

The conditions at the edge of the beam depend on its constraints. In the case of simply supported beam whose length is L, the vertical displacement at the beam ends are given as:

$$v(0,t) = v(L,t) = 0 \qquad (3.0)$$

$$v''(0,t) = v''(L,t) = 0 \qquad (3.1)$$

The derivative of above equation with respect to x

the initial conditions of the beam is given as

$$v(x,0) = 0 = v_{tt}(x,0) \qquad (3.2)$$

The non uniform prestress $N(x)$ is define as

$$N(x) = N_o\left(1 + \sin\frac{\pi x}{L}\right) \qquad (3.3)$$

where N_o is constant axial force

Two special states of equation (2.0) are considered. They are termed steady and harmonic variable loads problems.

4 First State of the Loads–steady Loads

4.1 The dynamic behavior of structural element acted upon by the two moving steady loads

The repeated concentrated mobile force $P_i(x,t)$ in equation (2.0) is given by

$$P_i(x,t) = P_1\delta(x - vt) + P_o\delta\{x - (vt + d)\}$$

$$0 \le t \le \frac{l-d}{v} \qquad (4.0)$$

P_1 P_c
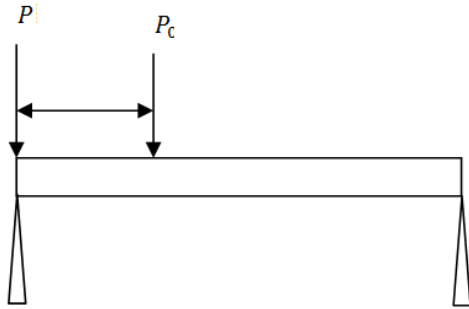
5 Approximate Analytical Solution

In this section, in order to compute the dynamic vibration of beam due to moving two loads, the non-homogeneous principal partial differential equation of order is solved by Fourier sine transformation method described by

$$w(j,t) = \int_0^L v(x,t) \sin \frac{j\pi x}{L} dx \qquad (5.0)$$

With the inverse

$$v(x,t) = \frac{2}{L} \sum_{j=1}^{\infty} w(j,t) \sin \frac{j\pi x}{L} dx \qquad (5.1)$$

Thus, substituting equations (2.1), (3.3) and (4.1) into the system of equation (2.0) the result is a non-homogeneous system of partial differential equation is given by

$$\frac{2}{L} \sum_{j=1}^{n} \left[\frac{EI}{4} \left(\frac{j\pi}{L} \right)^4 w(j,t) + \mu \ddot{w}(j,t) \right.$$

$$+ N_0 \left(1 + \sin \frac{\pi x}{L} \right) \left(\frac{j\pi}{L} \right)^2 w(j,t)$$

$$+ K w(j,t) + \varepsilon_o \dot{w}(i,j) \big] \sin \frac{i\pi x}{L}$$

$$= P_1 \delta(x - vt) + P_o \delta\{x - (vt + d)\} \qquad (5.2)$$

The value $w(j,t)$ in the equation above is evaluated by taking the orthogonality to the functions $\sin \frac{m\pi x}{L}$

Thus,

$$\int_0^L \frac{2}{L} \sum_{i=1}^n \left\{ \left(\left(\frac{EI}{4} \left(\frac{j\pi}{L} \right)^4 + N_0 \left(1 + \sin \frac{\pi x}{L} \right) \right. \right. \right.$$

$$\times \left. \left(\frac{j\pi}{L} \right)^2 + K \right) w(j,t)$$

$$+ \mu \ddot{w}(j,t) + \varepsilon_0 \dot{w}(j,t) \} \sin \frac{j\pi x}{L} \sin \frac{m\pi x}{L}$$

$$= \int_0^L P_1 \delta(x - vt) + P_o \delta\{x - (vt + d)\} \sin \frac{m\pi x}{L} \qquad (5.3)$$

The equation (5.3) above can be rearranged to take the form

$$b_1(j,m) \ddot{w}_i(t) + b_2(j,m) \dot{w}_i(t) + b_3(j,m) w_i(t)$$

$$= P_1 \sin \frac{m\pi ct}{L} + P_0 \sin \frac{m\pi ct}{L} + P_0 \sin \frac{m\pi d}{L} \qquad (5.4)$$

Where

$$b_1(j,m) = \mu I_1 \qquad (5.5\text{a})$$

$$b_2(j,m) = \varepsilon_o I_1 \qquad (5.5\text{b})$$

$$b_3(i,k) = \left(\frac{EI}{4} \left(\frac{j\pi}{L} \right)^4 + K \right) I_1$$

$$+ \left(\frac{j\pi}{L} \right)^2 N_0 (I_1 + I_2) \qquad (5.5\text{c})$$

Next, we take the Laplace transformation dynamic system of equation (5.4) above described by

$$(\sim) = \int_o^\infty (\cdot) e^{-st} dt \qquad (5.6)$$

In view of Laplace transform in equation (5.6), equation (5.4) becomes

$$(b_1(i,k) S^2 + b_2(i,k) S + b_3(i,k)) w_i(s)$$

$$= \left[a_1 \frac{\gamma_0}{S^2 + \gamma_o^2} + a_2 \frac{s}{S^2 + \gamma_o^2} \right] \qquad (5.7)$$

where

$$\gamma = \frac{m\pi ct}{L}, \quad \phi = \frac{m\pi d}{L} \quad a_1 = P_1 + P_0 \cos \phi,$$

$$a_2 = P_0 \sin \phi \tag{5.8}$$

Equation (5.7) is transformed to give

$$w_i(s) = \left[a_1 \frac{\gamma_0}{S^2 + \gamma_o^2} + a_2 \frac{s}{S^2 + \gamma_o^2} \right]$$

$$\times \frac{1}{(b_1(i,k)S^2 + b_2(i,k)S + b_3(i,k))} \tag{5.9}$$

which reduces to

$$w_i(s) = \frac{1}{(\eta_1 - \eta_2)} \left(\frac{1}{S - \eta_1} - \frac{1}{S - \eta_2} \right)$$

$$\times \left[a_1 \frac{\gamma_0}{S^2 + \gamma_o^2} + a_2 \frac{s}{S^2 + \gamma_o^2} \right] \tag{5.10}$$

Where

$$\eta_1 = \frac{-b_2}{2b_1} + \frac{(b_2^2 - 4b_1b_3)^{\frac{1}{2}}}{2b_1} \quad \text{and}$$

$$\eta_2 = \frac{-b_2}{2b_1} - \frac{(b_2^2 - 4b_1b_3)^{\frac{1}{2}}}{2b_1} \tag{5.11}$$

The following notations are obtained in view of equation Laplace inversion of (5.10) as follows:

$$g_1(s) = \frac{S}{S^2 + \gamma^2}, g_2(s) = \frac{\gamma}{S^2 + \gamma^2},$$

$$f_1(s) = \frac{\eta_1}{S - \eta_1} \quad \text{and} \quad f_2(s) = \frac{\eta_2}{S - \eta_2} \tag{5.12}$$

The convolution of equation (5.10) is defined by

$$f_s * g = \int_0^t f_i(t - u)g(u)du, \qquad i = 1,2 \tag{5.13}$$

From the convolution defined above, equation (5.10) is then express by

$$w_i(t) = P_p \left[\frac{e^{\eta_1 t}}{\eta_1} (a_2\tau_1 - a_1\tau_2) - \frac{e^{\eta_2 t}}{\eta_2} (a_2\tau_3 - a_1\tau_4) \right] \tag{5.14}$$

Where

$$P_p = \frac{1}{(\eta_1 + \eta_2)}$$

$$\tau_1 = \int_0^t e^{-\eta_1 u} \cos \gamma u\, du \qquad , \ \tau_2 = \int_0^t e^{-\eta_1 u} \sin \gamma u\, du$$

$$\tau_3 = \int_0^t e^{-\eta_2 u} \cos \gamma u\, du \qquad , \ \tau_4 = \int_0^t e^{-\eta_2 u} \sin \gamma u\, du \tag{5.15}$$

Equation (5.15) is evaluated using integration by part ones obtain the following

$$\tau_1 = \frac{\gamma}{(\gamma^2 + \eta_1^2)} \left(\frac{\eta_1}{\gamma} \left[1 - e^{\eta_1 t} \cos \gamma t \right] - e^{\eta_1 t} \sin \gamma t \right) \tag{5.16}$$

$$\tau_2 = \frac{\gamma}{(\gamma^2 + \eta_1^2)} \left(1 - e^{-\eta_1 t} \cos \gamma t - \frac{\eta_1}{\gamma} e^{-\eta_1 t} \sin \gamma t \right) \tag{5.17}$$

$$\tau_3 = \frac{\gamma}{(\gamma^2 + \eta_2^2)} \left(\frac{\eta_2}{\gamma} \left[1 - e^{\eta_2 t} \cos \gamma t \right] - e^{\eta_2 t} \sin \gamma t \right) \tag{5.18}$$

$$\tau_4 = \frac{\gamma}{(\gamma^2 + \eta_2^2)} \left(1 - e^{-\eta_2 t} \cos \gamma t - \frac{\eta_2}{\gamma} e^{-\eta_2 t} \sin \gamma t \right) \tag{5.19}$$

Substituting equations (5.14)-(5.19) into equation (5.14) yields

$$w_i(t) = P_p \left\{ \left(\frac{\gamma e^{\eta_1 t}}{\eta_1 (\gamma^2 + \eta_1^2)} (P_1 + P_0 \cos \phi) \right. \right.$$

$$\times \left(\frac{\eta_1}{\gamma} \left[1 - e^{\eta_1 t} \cos \gamma t \right] - e^{\eta_1 t} \sin \gamma t \right)$$

$$\left. - P_0 \sin \phi \left(1 - e^{-\eta_1 t} \cos \gamma t - \frac{\eta_1}{\gamma} e^{-\eta_1 t} \sin \gamma t \right) \right)$$

$$- \left(\frac{\gamma e^{\eta_2 t}}{\eta_2 (\gamma^2 + \eta_2^2)} (P_1 + P_0 \cos \phi) \right.$$

$$\times \left(\frac{\eta_2}{\gamma} \left[1 - e^{\eta_2 t} \cos \gamma t \right] - e^{\eta_2 t} \sin \gamma t \right)$$

$$\left. \left. + P_0 \sin \phi \left(1 - e^{-\eta_2 t} \cos \gamma t - \frac{\eta_2}{\gamma} e^{-\eta_2 t} \sin \gamma t \right) \right) \right\} \tag{5.20}$$

The equation (5.20) above is substituted into (5.1) which gives us the expression below

$$v(x,t) = \sum_{i=1}^{n} P_p \left\{ \left(\frac{\gamma e^{\eta_1 t}}{\eta_1 (\gamma^2 + \eta_1^2)} (P_1 + P_0 \cos \phi) \right. \right.$$

$$\times \left(\frac{\eta_1}{\gamma} \left[1 - e^{\eta_1 t} \cos \gamma t \right] - e^{\eta_1 t} \sin \gamma t \right)$$

$$- P_0 \sin \phi \left(1 - e^{-\eta_1 t} \cos \gamma t - \frac{\eta_1}{\gamma} e^{-\eta_1 t} \sin \gamma t \right) \right)$$

$$- \left(\frac{\gamma e^{\eta_2 t}}{\eta_2 (\gamma^2 + \eta_2^2)} (P_1 + P_0 \cos \phi) \right.$$

$$\left(\frac{\eta_2}{\gamma} \left[1 - e^{\eta_2 t} \cos \gamma t \right] - e^{\eta_2 t} \sin \gamma t \right)$$

$$\left. \left. + P_0 \sin \phi \left(1 - e^{-\eta_2 t} \cos \gamma t - \frac{\eta_2}{\gamma} e^{-\eta_2 t} \sin \gamma t \right) \right) \right\}$$

$$\times \sin \frac{m \pi x}{L} \qquad (5.21)$$

The expression (5.21) is the transverse displacement of the elastic structure under concentrated loads when P_1 and P_0 are not equal

If the loads are of the same value, then representation (5.21) gives

$$v(x,t) = \sum_{i=1}^{n} P_p P_0 \left\{ \left(\frac{\gamma e^{\eta_1 t}}{\eta_1 (\gamma^2 + \eta_1^2)} (1 + \cos \phi) \right. \right.$$

$$\times \left(\frac{\eta_1}{\gamma} \left[1 - e^{\eta_1 t} \cos \gamma t \right] - e^{\eta_1 t} \sin \gamma t \right)$$

$$- \sin \phi \left(1 - e^{-\eta_1 t} \cos \gamma t - \frac{\eta_1}{\gamma} e^{-\eta_1 t} \sin \gamma t \right) \right)$$

$$- \left(\frac{\gamma e^{\eta_2 t}}{\eta_2 (\gamma^2 + \eta_2^2)} (1 + \cos \phi) \right.$$

$$\times \left(\frac{\eta_2}{\gamma} \left[1 - e^{\eta_2 t} \cos \gamma t \right] - e^{\eta_2 t} \sin \gamma t \right)$$

$$+ \sin \phi \left(1 - e^{-\eta_2 t} \cos \gamma t - \frac{\eta_2}{\gamma} e^{-\eta_2 t} \sin \gamma t \right) \right) \right\} \times \sin \frac{m \pi x}{L} \tag{5.22}$$

If the loads are of the same value, and the space connecting the two loads is zero, solution (5.22) becomes

$$v(x,t) = \sum_{i=1}^{n} P_p P_0 \left(1 + \cos \phi \right) \left\{ \left(\frac{\gamma e^{\eta_1 t}}{\eta_1 (\gamma^2 + \eta_1^2)} \right) \right.$$

$$\times \left(\frac{\eta_1}{\gamma} \left[1 - e^{\eta_1 t} \cos \gamma t \right] - e^{\eta_1 t} \sin \gamma t \right)$$

$$- \left(\frac{\gamma e^{\eta_2 t}}{\eta_2 (\gamma^2 + \eta_2^2)} \right) \left(\frac{\eta_2}{\gamma} \left[1 - e^{\eta_2 t} \cos \gamma t \right] - e^{\eta_2 t} \sin \gamma t \right) \right\}$$

$$\times \sin \frac{m \pi x}{L} \tag{5.23}$$

6 Second State of the Loads–harmonic Variable Magnitude Loads

The dynamic behavior of structural element acted upon by the two moving harmonic variable magnitude loads.

Here, the moving force $P_i(x,t)$ is given as

$$P_i(x,t) = \sin \Omega t \left[P_1 \delta(x - vt) + P_o \delta \{ x - (vt + d) \} \right] \tag{6.0}$$

where all parameters are as defined as before. Thus in view of equation (2.0) taking into account (6.0) one obtains

$$\sum_{j=1}^{n} \left[\frac{EI}{4} \left(\frac{j\pi}{L} \right)^4 w(j,t) + \mu \ddot{w}(j,t) \right.$$

$$+ N_0 \left(1 + \sin \frac{\pi x}{L} \right) \left(\frac{j\pi}{L} \right)^2 w(j,t)$$

$$+ Kw(j,t) + \varepsilon_o \dot{w}(i,j) \left] \sin \frac{i\pi x}{L} \right.$$

$$= \cos \Omega t \left[P_1 \delta(x - vt) + P_o \delta \{ x - (vt + d) \} \right] \tag{6.1}$$

The above Equation (6.1) is the principal equation relating the motion of elastic beam under the influence of two forces of varying magnitude. The section also follows the same approach in the case I, the transverse motion $v_a(x,t)$ of beam acted upon by the action of variable magnitude mobile force can be written as

$$z_h(h,t) = \int_0^L v_a(x,t) \sin \frac{h\pi x}{L} dx \qquad (6.2)$$

With the inverse

$$v_a(x,t) = \sum_{h=1}^{\infty} z_h(h,t) \sin \frac{h\pi x}{L} dx \qquad (6.3)$$

Repeating or following the same pattern as in the previous section, one obtains

$$\sum_{h=1}^{\infty} \{b_1(h,k)\ddot{z}_h(t) + b_2(h,k)\dot{z}_h(t) + b_3(h,k)z_h(t)\}$$

$$= P_o \sin \Omega t \sin \frac{k\pi x_o}{L} + P_o \sin \Omega t \sin \frac{k\pi \beta \alpha t}{L} \qquad (6.4)$$

Putting into consideration h^{th} particle of the whole system of moving loads we have

$$b_1(h,k)\ddot{z}_h(t) + b_2(h,k)\dot{z}_h(t) + b_3(h,k)z_h(t)$$

$$= P_o \sin \Omega t \sin \frac{k\pi x_o}{L} + P_o \sin \Omega t \sin \frac{k\pi \beta \alpha t}{L} \qquad (6.5)$$

Subjecting equation (6.5) as defined previously yields

$$z_h(S) = P_p \left\{ \frac{1}{\Psi_1} \left[g_o \left(\frac{\omega_1}{S^2 + \omega_1^2} \cdot \frac{\Psi_1}{S - \Psi_1} + \frac{\omega_2}{S^2 + \omega_2^2} \cdot \frac{\Psi_1}{S - \Psi_1} \right) \right. \right.$$

$$\left. - a_o \left(\frac{S}{S^2 + \omega_1^2} \cdot \frac{\Psi_1}{S - \Psi_1} + \frac{S}{S^2 + \omega_2^2} \cdot \frac{\Psi_1}{S - \Psi_1} \right) \right]$$

$$- \frac{1}{\Psi_2} \left[g_o \left(\frac{\omega_1}{S^2 + \omega_1^2} \cdot \frac{\Psi_2}{S - \Psi_2} + \frac{\omega_2}{S^2 + \omega_2^2} \cdot \frac{\Psi_2}{S - \Psi_2} \right) \right.$$

$$\left. \left. - a_o \left(\frac{S}{S^2 + \omega_1^2} \cdot \frac{\Psi_2}{S - \Psi_2} + \frac{S}{S^2 + \omega_2^2} \cdot \frac{\Psi_2}{S - \Psi_2} \right) \right] \right\} \qquad (6.6)$$

Where

$$\Psi_1 = \frac{-b_2 + \left(b_2^2 - 4b_1 b_3\right)^{\frac{1}{2}}}{2b_1} \quad \text{and} \quad \Psi_2 = \frac{-b_2 - \left(b_2^2 - 4b_1 b_3\right)^{\frac{1}{2}}}{2b_1}$$

$$\omega_1 = \Omega + \frac{k\pi\gamma}{L} \quad \text{and} \quad \omega_2 = \Omega - \frac{k\pi\gamma}{L} \tag{6.7}$$

By following the same procedure as in equation (5.10) and the convolution theory, equation (6.6) becomes

$$y_h(t) = P_p \left\{ \frac{e^{\Psi_1 t}}{\Psi_1} \left\langle \frac{a_0 \omega_1}{(\omega_1^2 + \Psi_1^2)} \right. \right.$$

$$\times \left(1 - e^{-\Psi_1 t} \cos \omega_1 t - \frac{\Psi_1}{\omega_1} e^{-\Psi_1 t} \sin \omega_1 t \right)$$

$$+ \frac{a_0 \omega_2}{(\omega_2^2 + \Psi_1^2)} \left(1 - e^{-\Psi_1 t} \cos \omega_2 t - \frac{\Psi_1}{\omega_2} e^{-\Psi_1 t} \sin \omega_2 t \right) \right]$$

$$+ \left(\frac{b_0 \omega_1}{(\omega_1^2 + \Psi_1^2)} \left(\frac{\Psi_1}{\omega_1} \left[1 - e^{\Psi_1 t} \cos \omega_1 t \right] - e^{\Psi_1 t} \sin \omega_1 t \right) \right.$$

$$\left. \left. - \frac{b_0 \omega_2}{(\omega_2^2 + \Psi_1^2)} \left(\frac{\Psi_1}{\omega_2} \left[1 - e^{\Psi_1 t} \cos \omega_2 t \right] - e^{\Psi_1 t} \sin \omega_2 t \right) \right) \right\rangle$$

$$- \frac{e^{\Psi_2 t}}{\Psi_2} \left\langle \frac{a_0 \omega_1}{(\omega_1^2 + \Psi_2^2)} \left(1 - e^{-\Psi_2 t} \cos \omega_1 t - \frac{\Psi_2}{\omega_1} e^{-\Psi_2 t} \sin \omega_1 t \right) \right.$$

$$+ \frac{a_0 \omega_2}{(\omega_2^2 + \Psi_2^2)} \left(1 - e^{-\Psi_2 t} \cos \omega_2 t - \frac{\Psi_2}{\omega_2} e^{-\Psi_2 t} \sin \omega_2 t \right) \right]$$

$$+ \left(\frac{b_0 \omega_1}{(\omega_1^2 + \Psi_2^2)} \left(\frac{\Psi_2}{\omega_1} \left[1 - e^{\Psi_2 t} \cos \omega_1 t \right] - e^{\Psi_2 t} \sin \omega_1 t \right) \right.$$

$$\left. \left. - \frac{b_0 \omega_2}{(\omega_2^2 + \Psi_2^2)} \left(\frac{\Psi_2}{\omega_2} \left[1 - e^{\Psi_2 t} \cos \omega_2 t \right] - e^{\Psi_2 t} \sin \omega_2 t \right) \right) \right\rangle \right\} \tag{6.8}$$

which on inversion yields

$$v_a(x,t) = \sum_{m=1}^{n} P_p \left\{ \frac{e^{\Psi_1 t}}{\Psi_1} \left\langle \frac{a_0 \omega_1}{(\omega_1^2 + \Psi_1^2)} \right. \right.$$

$$\times \left(1 - e^{-\Psi_1 t} \cos \omega_1 t - \frac{\Psi_1}{\omega_1} e^{-\Psi_1 t} \sin \omega_1 t \right)$$

$$+ \frac{a_0 \omega_2}{(\omega_2^2 + \Psi_1^2)} \left(1 - e^{-\Psi_1 t} \cos \omega_2 t - \frac{\Psi_1}{\omega_2} e^{-\Psi_1 t} \sin \omega_2 t \right) \Bigg]$$

$$+ \left(\frac{b_0 \omega_1}{(\omega_1^2 + \Psi_1^2)} \left(\frac{\Psi_1}{\omega_1} \left[1 - e^{\Psi_1 t} \cos \omega_1 t \right] - e^{\Psi_1 t} \sin \omega_1 t \right) \right.$$

$$\left. - \frac{b_0 \omega_2}{(\omega_2^2 + \Psi_1^2)} \left(\frac{\Psi_1}{\omega_2} \left[1 - e^{\Psi_1 t} \cos \omega_2 t \right] - e^{\Psi_1 t} \sin \omega_2 t \right) \right) \Bigg\rangle$$

$$- \frac{e^{\Psi_2 t}}{\Psi_2} \left\langle \frac{a_0 \omega_1}{(\omega_1^2 + \Psi_2^2)} \left(1 - e^{-\Psi_2 t} \cos \omega_1 t - \frac{\Psi_2}{\omega_1} e^{-\Psi_2 t} \sin \omega_1 t \right) \right.$$

$$+ \frac{a_0 \omega_2}{(\omega_2^2 + \Psi_2^2)} \left(1 - e^{-\Psi_2 t} \cos \omega_2 t - \frac{\Psi_2}{\omega_2} e^{-\Psi_2 t} \sin \omega_2 t \right) \Bigg]$$

$$+ \left(\frac{b_0 \omega_1}{(\omega_1^2 + \Psi_2^2)} \left(\frac{\Psi_2}{\omega_1} \left[1 - e^{\Psi_2 t} \cos \omega_1 t \right] - e^{\Psi_2 t} \sin \omega_1 t \right) \right.$$

$$\left. - \frac{b_0 \omega_2}{(\omega_2^2 + \Psi_2^2)} \left(\frac{\Psi_2}{\omega_2} \left[1 - e^{\Psi_2 t} \cos \omega_2 t \right] - e^{\Psi_2 t} \sin \omega_2 t \right) \right) \Bigg\rangle \Bigg\}$$

$$\times \sin \frac{m \pi x}{L} \tag{6.9}$$

7 Analysis for Resonance in the Dynamic System

The resonance phenomenon of our dynamic system is analyzed at this junction. There are situations whereby vibrations go beyond certain limit. Taking a close look at equation (5.23), it is observe that the elastic beam resting on elastic foundation will experience resonance effects whenever

$$\eta_1 = \eta_2, \qquad \eta_1^2 = -\gamma^2 \qquad or \qquad \eta_2^2 = -\gamma^2 \tag{7.1}$$

Equation (6.10) follows similar trend for the same beam when harmonic variable magnitude moving loads is considered in place of steady load.

$$\Psi_1 = \Psi_2, \qquad \Psi_1^2 = -\omega_1^2 \qquad or \qquad \Psi_2^2 = -\omega_1^2 \quad \text{and also}$$

$$\Psi_1^2 = -\omega_2^2 \qquad or \qquad \Psi_2^2 = -\omega_2^2 \tag{7.2}$$

8 Comments on the Numerical Results

We shall illustrate the analysis proposed in this paper by considering a beam by adopting beam parameters and material properties defined in Oni and Ogunyebi 2008 [11]. These properties are of modulus of elasticity $E = 2.109 \times 10^9 \, kg/m^2$, moment of inertia $I = 2.87698 \times 10^{-3} m^4$ and the mass per unit length of the beam $\mu = 2758.291 kg/m$. The beam span in this study is taking to be L= 50.192m. Foundation moduli of the beam are taken in range of 2 x 10^6 N/m^3 and 4 x 10^6 N/ m^3. The range of prestress N_0 is in between 0 N/ m^3 and 6 x10^4 N / m^3. The results are as shown on the various graphs below for the simply supported boundary condition so far considered.
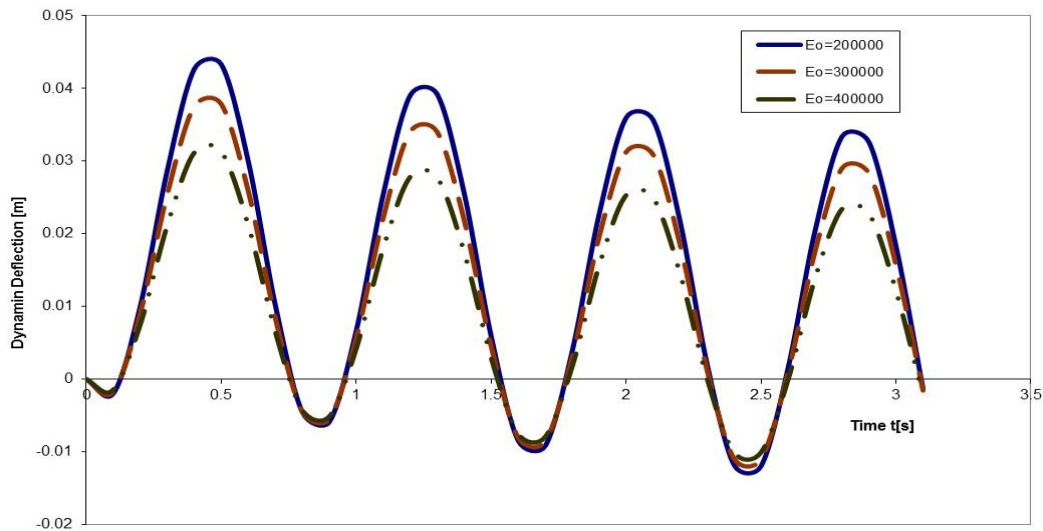
Fig. 1. Dynamic deflection for Euler–Bernoulli beam under two steady concentrated moving loads with different values of foundation moduli E_0

Figs. 1 and 6 illustrate the response of Euler–Bernoulli beam under the two travelling loads are of constant and variable magnitude respectively for different values of foundation modulus E_0. It is observed that an increase in the foundation modulus E_0 resulted to decrease in the amplitude of vibration.

Figs. 2 and 7 depict the influence of prestress N_0 on the deflection profile of the beam in both cases of steady and variable moving loads respectively. It is evidenced that higher values of prestress N_0 reduce the

pulsation of the beams. Figs. 3 and 8 depict that as the distance d apart of the two loads P_0 and P_1 increases, the deflection of Euler–Bernoulli beam decreases.
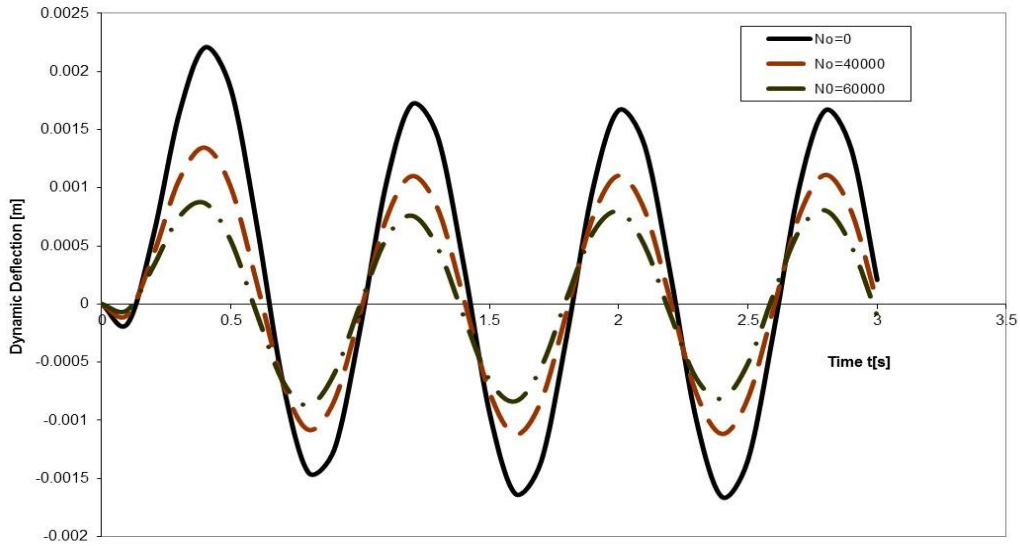
Fig. 2. Dynamic deflection for Euler–Bernoulli beam under two steady concentrated moving loads with different values of prestress N_0
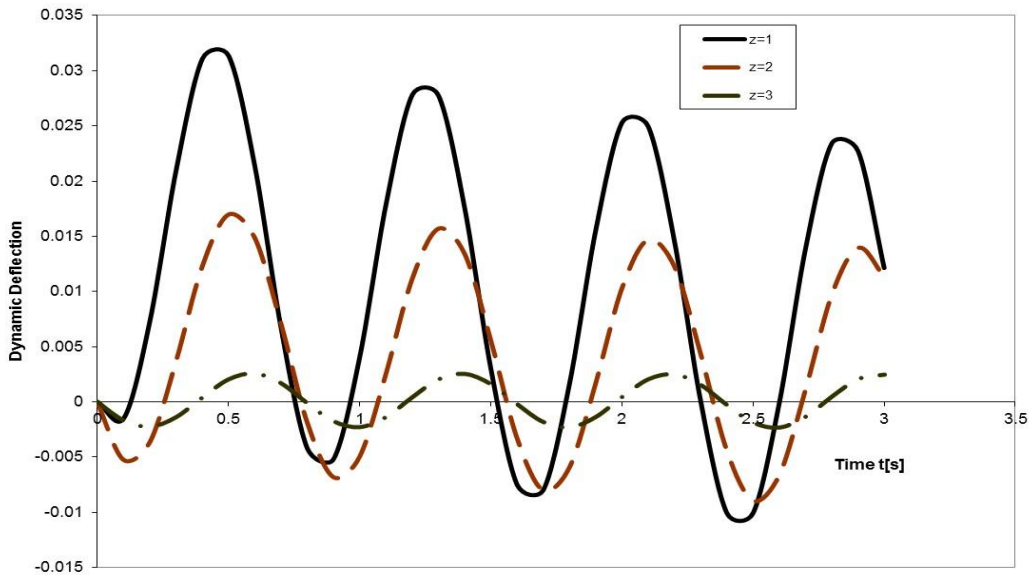
Fig. 3. Dynamic deflection for Euler–Bernoulli beam under two steady concentrated moving loads with various space or gap d apart of the loads
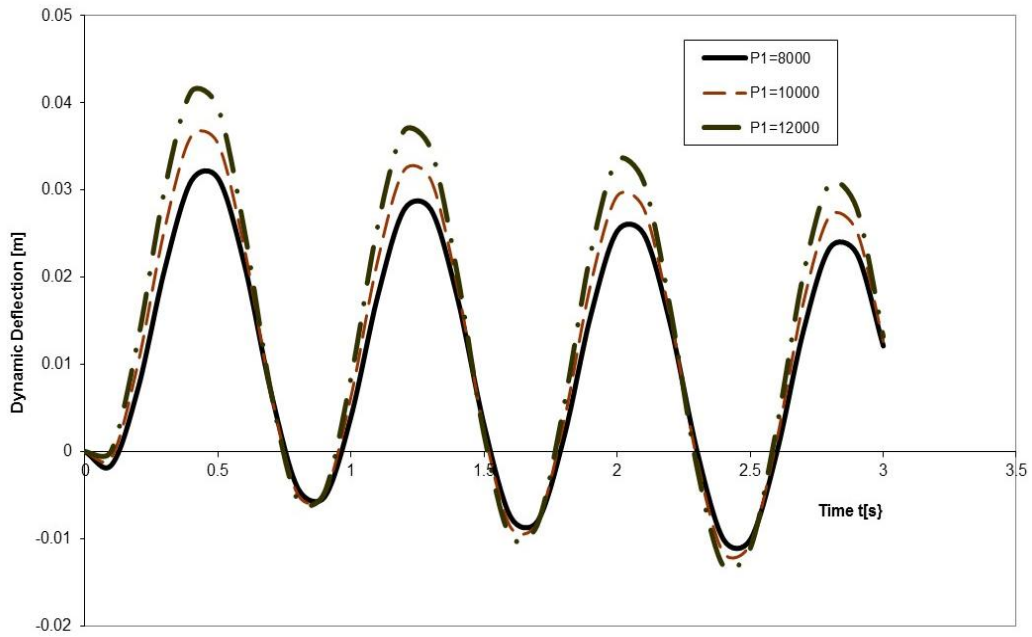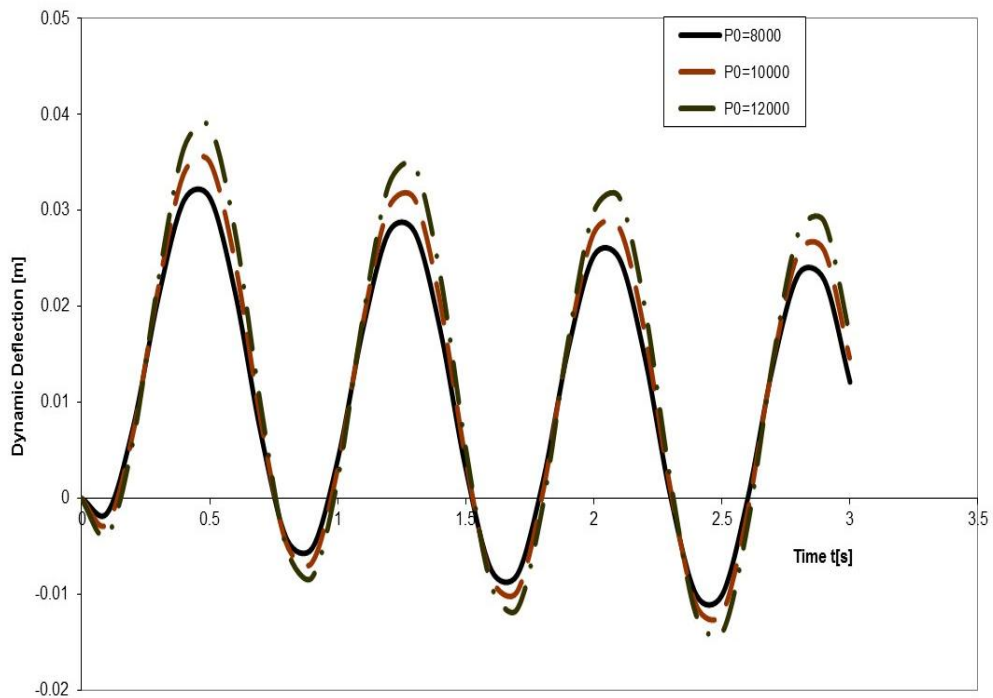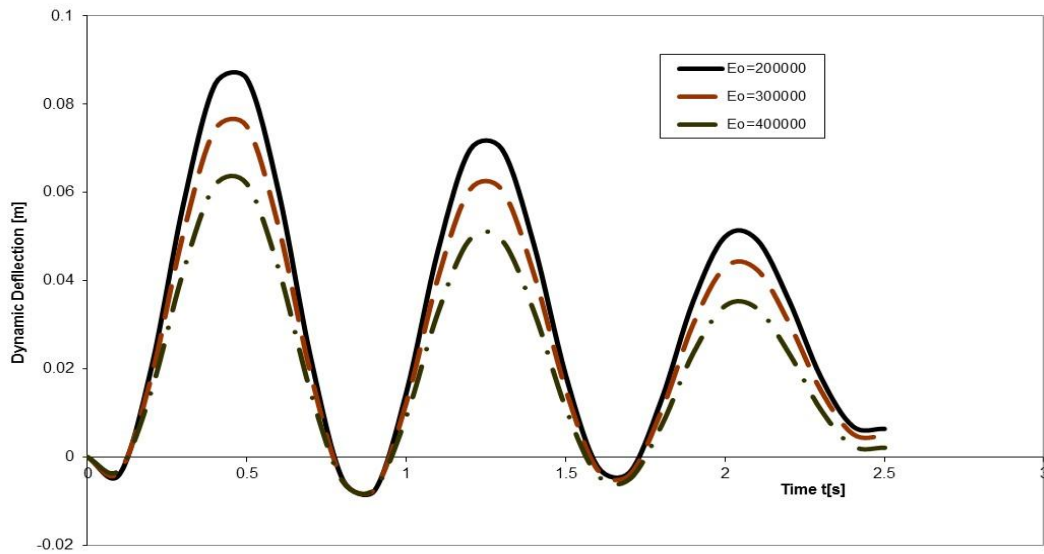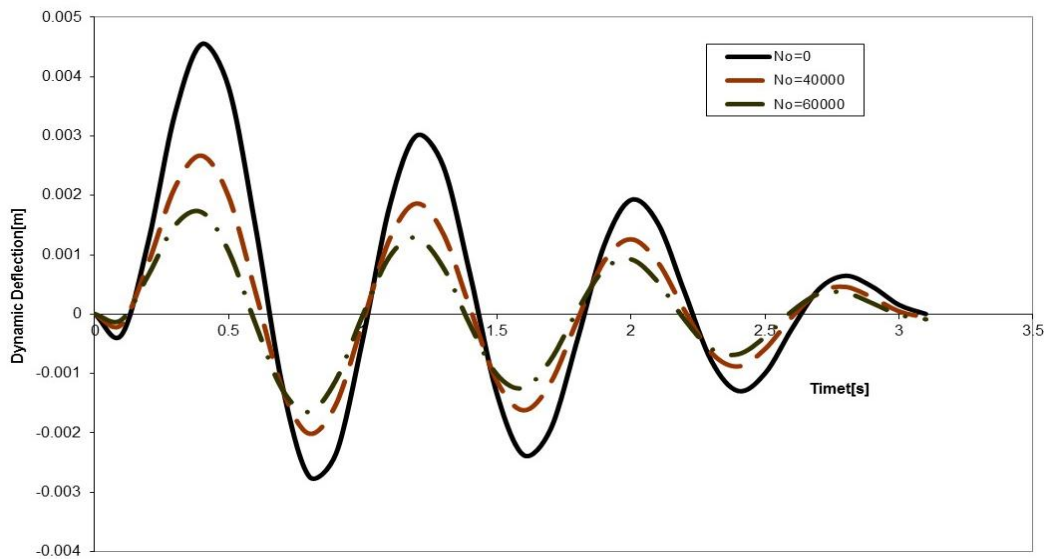
Fig. 4. Dynamic deflection for Euler–Bernoulli beam under two steady concentrated moving loads with different values of the loads P_1

Fig. 5. Dynamic deflection for Euler–Bernoulli beam under two steady concentrated moving loads with different values one of the loads P_0

Fig. 6. Dynamic deflection for Euler–Bernoulli beam under two Harmonic variable magnitude moving loads with different values of foundation moduli E_0

Fig. 7. Dynamic deflection for Euler–Bernoulli beam under two Harmonic variable magnitude moving loads with different values of prestress N_0

From the graphs in Figs. 4 and 9 display the effect of various magnitudes of travelling loads P_1 on the Euler–Bernoulli beam when P_1 is steady and variable magnitude respectively. The graphs show that the response amplitude increases as the value of the P_1 increases. Similarly, Figs. 5 and 10 display the effect of

various magnitude of travelling loads P_0 on the Euler–Bernoulli beam when P_0 is steady and variable magnitude respectively. The graphs show that the response amplitude increases as the value of the P_0 increases.
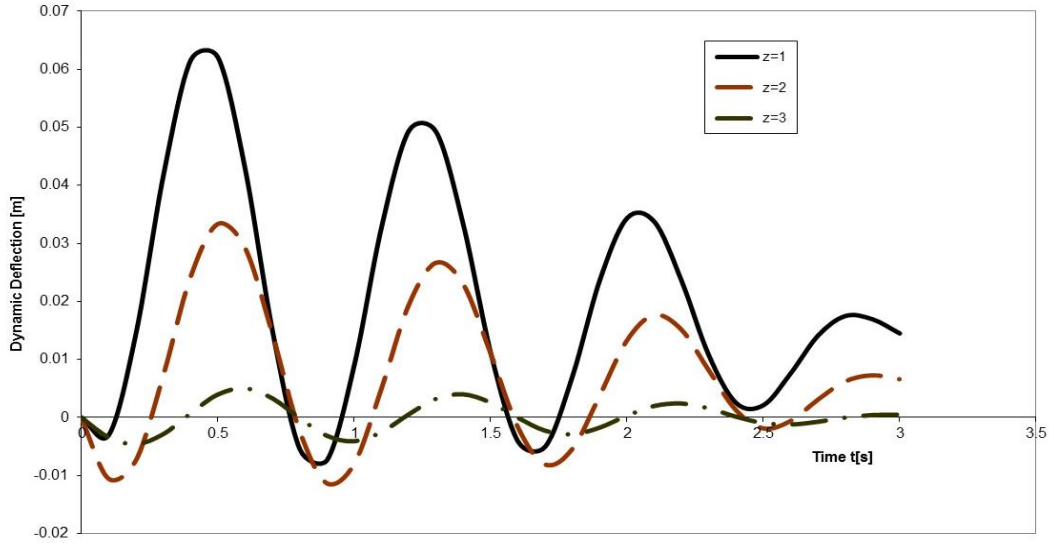

Fig. 8. Dynamic deflection for Euler–Bernoulli beam under two Harmonic variable magnitude moving loads with various space or gap d apart of the loads
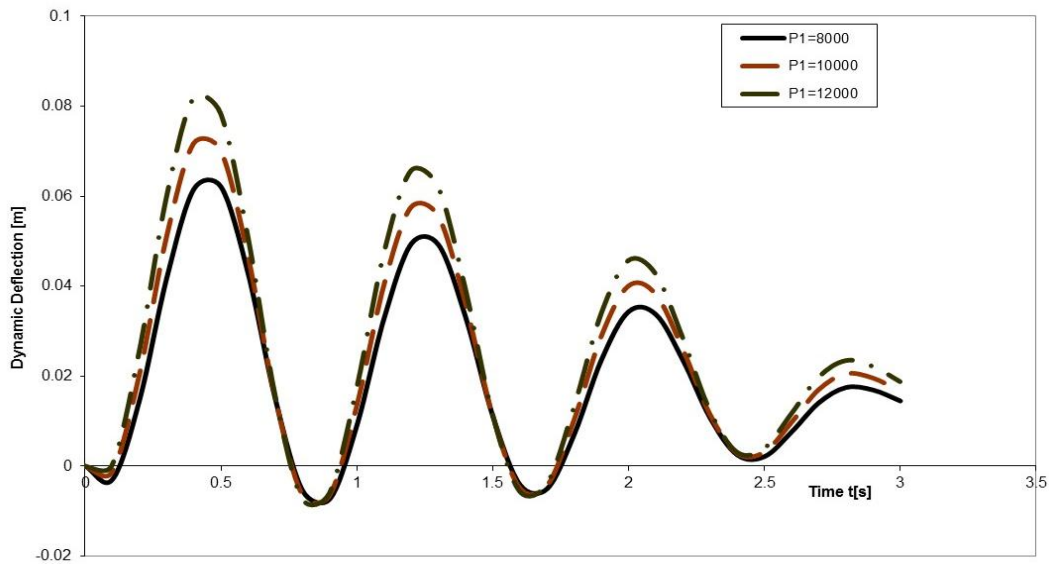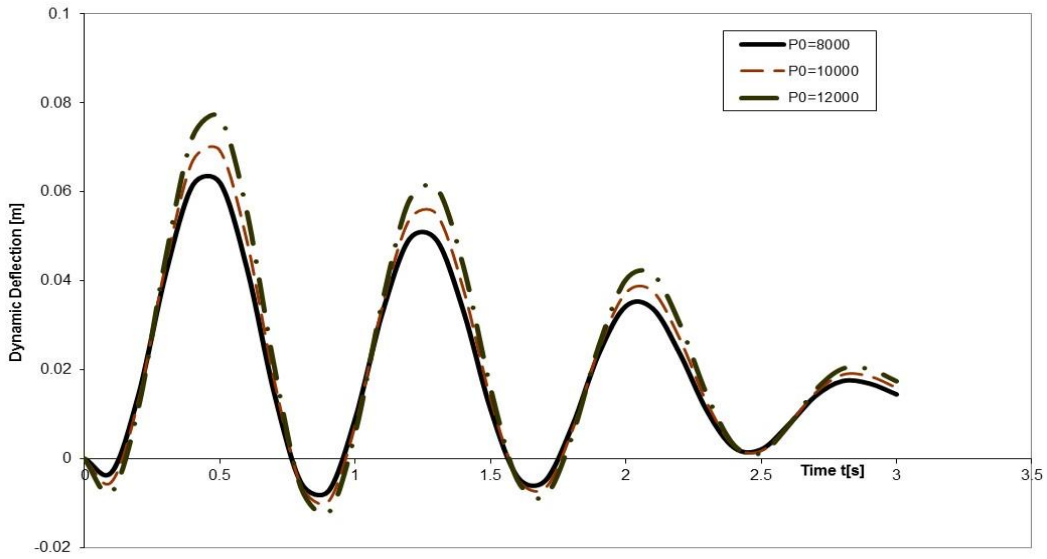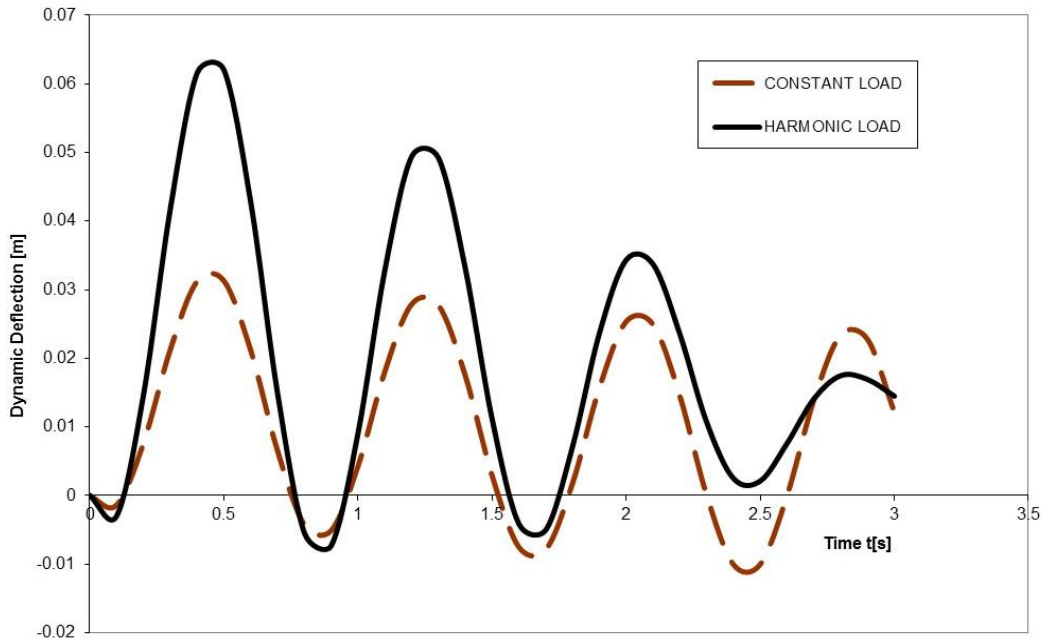

Fig. 9. Dynamic deflection for Euler–Bernoulli beam under two Harmonic variable magnitude concentrated moving loads with different values of the loads P_1

Fig. 10. Dynamic deflection for Euler–Bernoulli beam under two Harmonic variable magnitudes concentrated moving loads with different values one of the loads P_0

Fig. 11. Comparison of the amplitude of vibration Euler–Bernoulli beam subjected to steady and Harmonic variable magnitudes concentrated moving loads with respect to time

The Comparison of the amplitude of vibration Euler–Bernoulli beam subjected to steady and Harmonic variable magnitudes concentrated moving loads with respect to time is depicted in Fig. 11. The response amplitude of variable magnitude moving load is higher than that of the constant magnitude moving load.

9 Conclusion

(i) The dynamic response amplitudes of beam decreases as the distance d between the loads increases when the values of the beam parameters, prestressed N_0 and foundation modulus E_0 are fixed.

(ii) Increase in the values of foundation modulus E_0 reduces the displacement response of the elastic beam for fixed value prestressed N_0.

(iii) As the value of prestressed N_0 increases, the displacement response of beam decreases for fixed values of foundation modulus, E_0, the distance d between the loads.

(iv) Increase in the values of the beam parameters namely foundation modulus E_0, and prestressed N_0 reduce amplitude of vibrating system involving beam under the actions of concentrated moving two loads increases and the risk of resonance is sufficiently reduced.

Competing Interests

Author has declared that no competing interests exist.

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