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Generalized $\mathcal{I}p$-Closed Sets in Ideal Topological Spaces

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Authors' contributions

This work was carried out in collaboration between both authors. Author PE developed the concept of generalized closed sets in ideal topological spaces and proved several foundational results. Author SR investigated the connections between the newly found generalized closed set and other generalized closed sets in ideal topological spaces, provided the mathematical proofs for several key results, and considered examples and counterexamples to illustrate the concepts. Both authors have read and approved the final version of the manuscript.

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ABSTRACT

The main focus of this study is to introduce a new category of generalized closed sets, referred to as $\mathcal{I}p$-closed sets, within the framework of ideal topological spaces. By using a few instances, we demonstrate $\mathcal{I}p$-closed sets and establish some fundamental properties of $\mathcal{I}p$-closed sets. We also investigate the relationship between $\mathcal{I}p$-closed sets and other classes of generalized closed sets in ideal topological spaces, such as $\mathcal{I}g$-closed sets, $\alpha\mathcal{I}g$-closed sets, and $\mathcal{I}rg$-closed sets. Then, we focus on the topological implications of $\mathcal{I}p$-closed sets and investigate how they relate to the concepts of $\mathcal{I}p$-continuous map, $\mathcal{I}p$-irresolute map, and a strongly $\mathcal{I}p$-continuous map. First and foremost, we define the $\mathcal{I}p$-continuous map, investigate the behavior of $\mathcal{I}p$-continuous map with respect to $\mathcal{I}p$-closed sets, and derive several important properties of $\mathcal{I}p$-continuous map.

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Further, we studied their relationships with other classes of continuous maps in ideal topological spaces. Nevertheless, we defined the definitions of $\mathcal{I}p$-irresolute maps and strongly $\mathcal{I}p$-continuous maps in ideal topological spaces. We explored the connections with the notions of $\mathcal{I}p$-continuous map, $\mathcal{I}p$-irresolute map, and a strongly $\mathcal{I}p$-continuous map. Our results provide new insights into the study of ideal topological spaces.

Keywords: Preopen set; ideals; $\mathcal{I}p$-closed set; $\mathcal{I}p$-continuous map; $\mathcal{I}p$-irresolute map.

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1 INTRODUCTION

Today's mathematics incorporates topological concepts into nearly every discipline. It has grown to be an effective tool for mathematical research. Levine[1] made a groundbreaking contribution to topology in 1970 by introducing the concept of generalized closed sets, also known as g-closed sets, within a topological space. After that, several mathematicians turned their attention to various forms of topology by finding new types of generalized closed sets[2, 3, 4].

Topological ideals have been a subject of investigation since early 1930, paving the way for several important discoveries in the field[5, 6]. The study of ideals in topological spaces was initiated almost fifty years ago by Kuratowski[7] and Vaidyanathaswamy[8], and since then, researchers have been actively exploring the use of topological ideals to extend basic concepts in general topology[9, 10, 11]. In the field of ideal topological spaces, Dontchev et al.[12] were the pioneers of the concept of $\mathcal{I}g$-closed sets. Navaneethakrishnan and Sivaraj[13] made significant contributions to the field of ideal topological spaces by introducing the concept of $\mathcal{I}rg$-closed sets. Further, the study of $\alpha\mathcal{I}g$ sets in ideal topological spaces was initiated by Maragathavalli and Vinothini[14].

In ideal topological spaces, the above-mentioned closed sets are defined as follows:

A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be

1. $\mathcal{I}g$-closed set[12] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open.
2. $\mathcal{I}rg$-closed set[15] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is regular open.
3. $\alpha\mathcal{I}g$-closed set [14] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is α-open.

Among their various research endeavors, Jankovi and Hamlett[16] looked into the notion of continuous maps in ideal topological spaces. Following that, several researchers came up with new types of continuous maps in ideal topological spaces, which included $\mathcal{I}g$-continuous maps, $\mathcal{I}rg$-continuous maps, and $\alpha\mathcal{I}g$-continuous maps.

In ideal topological spaces, the above-mentioned continuous maps are defined as follows:
A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is called

1. $\mathcal{I}g$-continuous[17] if $f^{-1}(V)$ is $\mathcal{I}g$-closed in X for every closed set V of Y.
2. $\mathcal{I}rg$-continuous[15] if $f^{-1}(V)$ is $\mathcal{I}rg$-closed in X for every closed set V of Y.
3. $\alpha\mathcal{I}g$-continuous[14] if $f^{-1}(V)$ is $\alpha\mathcal{I}g$-closed in X for every closed set V of Y.

The objective of this paper is to provide a detailed analysis of a newly defined set of generalized closed sets in ideal topological spaces known as $\mathcal{I}p$-closed sets. This paper will conduct an in-depth examination of the fundamental properties of this new class of generalized closed sets, and explore its connections with other types of generalized closed sets in ideal topological spaces. We defined the $\mathcal{I}p$-continuous map with an illustrated example. We have explored various properties of the new type of $\mathcal{I}p$-continuous map in this study, shedding light on its behavior in ideal topological spaces. Further, we defined the $\mathcal{I}p$-irresolute map and the strongly $\mathcal{I}p$-continuous map with some corresponding examples and investigated some significant properties.

2 MATERIALS AND METHODS

Definition 2.1. Let (X, τ) be a topological space. An ideal $\mathcal{I}$ on X is a nonempty family of subsets of X that satisfies the following two conditions:

1. $A \in \mathcal{I}$ and $B \subseteq A$ implies $B \in \mathcal{I}$.
2. $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$.

If $\mathcal{I}$ is an ideal on X, then $(X, \tau, \mathcal{I})$ is called an ideal topological space.

Suppose (X, τ) is a topological space and $\mathcal{I}$ is an ideal on X. Then, let $P(X)$ denote the set of all subsets of X. In the context of τ and $\mathcal{I}$, we can define a set operator $(.)^* : P(X) \rightarrow P(X)$ as a local function[7] of A. This function is defined as follows: for $A \subseteq X$, $A^*(\mathcal{I}, \tau)$ = $\{x \in X \mid U \cap A \notin \mathcal{I}$ for every $U \in \tau(x)\}$ where $\tau(x)$ = $\{U \in \tau \mid x \in U\}$. we use a Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(\mathcal{I}, \tau)$, which we call the $*$-topology, and it is finer than τ. The operator is defined as $cl^*(A) = A \cup A^*(\mathcal{I}, \tau)$[18]. To avoid any confusion, we will use A^* to refer to $A^*(\mathcal{I}, \tau)$ and τ^* to refer to $\tau^*(\mathcal{I}, \tau)$ when there is no possibility of ambiguity.

Definition 2.2. Let A be a subset of the topological space (X, τ). Then A is referred to as,

1. preopen set[19] if $A \subseteq int(cl(A))$.
2. α-open set[20] if $A \subseteq int(cl(int(A)))$.
3. regular open set[21] if $A = int(cl(A))$.

Lemma 2.1. *Given an ideal topological space $(X, \tau, \mathcal{I})$ and subsets A, B of X, the following properties are satisfied:[16]*

1. $A \subseteq B = A^* \subseteq B^*$,
2. $A^* = cl(A^*) \subseteq cl(A)$,
3. $(A^*)^* \subseteq A^*$,
4. $(A \cup B)^* = A^* \cup B^*$,
5. $(A \cap B)^* \subseteq A^* \cap B^*$.

Utilizing the materials mentioned above, we have developed a novel form of generalized closed set in ideal topological spaces known as $\mathcal{I}p$-closed sets. We can explore other properties by employing these sets.

3 RESULTS AND DISCUSSION

Definition 3.1. In an ideal topological space $(X, \tau, \mathcal{I})$, a subset A is known as a $\mathcal{I}p$-closed set if $A^* \subseteq U$ whenever $A \subseteq U$ and U is preopen.

Example 3.1.

1. *Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$. Thus, the preopen sets are X, $\emptyset$, $\{a\}$, $\{b\}$, $\{a, b\}$. As a result, the $\mathcal{I}p$-closed sets are X, $\emptyset$, $\{a\}$, $\{c\}$, $\{a, c\}$, $\{b, c\}$.*
2. *Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. Thus, the preopen sets are X, $\emptyset$, $\{a\}$, $\{a, b\}$, $\{a, c\}$. As a result, the $\mathcal{I}p$-closed sets are X, $\emptyset$, $\{b\}$, $\{c\}$, $\{b, c\}$.*
3. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{a, d\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. Thus, the preopen sets are X, $\emptyset$, $\{a\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{a, b, c\}$, $\{a, c, d\}$, $\{a, b, d\}$. As a result, the $\mathcal{I}p$-closed sets are X, $\emptyset$, $\{b\}$, $\{c\}$, $\{d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$, $\{b, c, d\}$.*

Remark 3.1. The fact that every closed set is a $\mathcal{I}p$-closed set can be observed as follows: Consider a closed set A and a preopen set U such that $A \subseteq U$. Then, $A^* \subseteq U$. However, it should be noted that not all $\mathcal{I}p$-closed sets are closed sets, as exemplified in examples 1 and 3.

Definition 3.2. In an ideal topological space $(X, \tau, \mathcal{I})$, a subset A is classified as a $\mathcal{I}p$-open set if its complement, A^c, is a $\mathcal{I}p$-closed set.

Proposition 3.1. *Let A be an $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$, and assume $B \subseteq A$. It follows that B is also an $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$.*

Proof. Suppose A is a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$. Then for any preopen set U containing A, we have $A^* \subseteq U$. Additionally, if $B \subseteq A$, then $B^* \subseteq A^*$. As a consequence, if $B \subseteq U$ where U is preopen, then $B^* \subseteq A^* \subseteq U$. Therefore, B is also a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$. □

Lemma 3.2. *In an ideal topological space, every $\mathcal{I}p$-closed set is a $\mathcal{I}g$-closed set.*

Proof. Let $A \subseteq U$ and U is open. Clearly every open set is a preopen set, it follows that U is also preopen. As A is a $\mathcal{I}p$-closed set, $A^* \subseteq U$, which implies that, A is a $\mathcal{I}g$-closed set. □

It is worth noting that the inverse of the lemma mentioned above does not hold in general. An example that demonstrates this fact is as follows:

Example 3.3. *Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Following that, the $\mathcal{I}g$-closed sets are X, $\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$ and the $\mathcal{I}p$-closed sets are X, $\emptyset$, $\{a\}$, $\{b\}$, $\{a, b\}$, $\{b, c\}$. In view of the fact that $\{c\}$ is a $\mathcal{I}g$-closed set but not a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$.*

Lemma 3.4. *In an ideal topological space, every $\mathcal{I}p$-closed set is a $\alpha\mathcal{I}g$-closed set.*

Proof. Let $A \subseteq U$ and U is α-open. Clearly every α-open set is a preopen set, it follows that U is also preopen. As A is a $\mathcal{I}p$-closed set, $A^* \subseteq U$, which implies that A is a $\alpha\mathcal{I}g$-closed set. □

It is worth noting that the inverse of the lemma mentioned above does not hold in general. An example that demonstrates this fact is as follows:

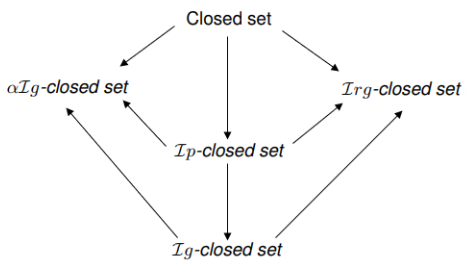
Example 3.5. *Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{b\}, \{a, c\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Following that, the $\alpha\mathcal{I}g$-closed sets are X, $\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$ and the $\mathcal{I}p$-closed sets are X, $\emptyset$, $\{b\}$, $\{a, c\}$. In view of the fact that $\{a, b\}$ is a $\alpha\mathcal{I}g$-closed set but not a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$.*

Lemma 3.6. *In an ideal topological space, every $\mathcal{I}p$-closed set is a $\mathcal{I}rg$-closed set.*

Proof. Let $A \subseteq U$ and U is regular-open. Clearly every regular-open set is a preopen set it follows that U is also preopen. As A is a $\mathcal{I}p$-closed set, $A^* \subseteq U$, which implies that A is a $\mathcal{I}rg$-closed set. □

It is worth noting that the inverse of the lemma mentioned above does not hold in general. An example that demonstrates this fact is as follows:

Example 3.7. *Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Following that, the $\mathcal{I}rg$-closed sets are X, $\emptyset$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$ and the $\mathcal{I}p$-closed sets are X, $\emptyset$, $\{c\}$, $\{a, c\}$, $\{b, c\}$. In view of the fact that $\{a, b\}$ is a $\mathcal{I}rg$-closed set but not a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$.*



The interplay between the recently uncovered $\mathcal{I}p$-closed sets and some of the other closed sets in ideal topological spaces is demonstrated in the above diagram.

Proposition 3.2. *Let A and B be two $\mathcal{I}p$-closed sets in the ideal topological space $(X, \tau, \mathcal{I})$. Then,*

1. *$A \cup B$ is a $\mathcal{I}p$-closed set.*
2. *$A \cap B$ is a $\mathcal{I}p$-closed set.*

Proof.

1. Let U be a preopen subset of X that contains the union of sets A and B (i.e., $A \cup B \subseteq U$). Then, either $A \subseteq U$ or $B \subseteq U$. Since A and B are both $\mathcal{I}p$-closed, we have that either A^* is a subset of U or B^* is a subset of U. Therefore, their union, $A^* \cup B^*$= $(A \cup B)^*$, is a subset of U, and thus $A \cup B$ is a $\mathcal{I}p$-closed set contained in U.
2. Assume A and B are $\mathcal{I}p$-closed sets in $(X, \tau, \mathcal{I})$, and let U be a preopen set such that A and B are subsets of U. Then, we have $A^* \subseteq U$ and $B^* \subseteq U$, since A and B are both subsets of U. Moreover, since $A \cap B$ is a subset of A, it follows that $A \cap B$ is also a subset of U. Similarly, $A \cap B$ is a subset of B, so $A \cap B$ is also a subset of U. Furthermore, $A^* \cap B^*$ is a subset of A^* and B^*, and since both A^* and B^* are subsets of U, then $A^* \cap B^*$ is also a subset of U. Consequently, we have $(A \cap B)^* \subseteq A^* \cap B^* \subseteq U$, whenever $A \cap B \subseteq U$ and U is preopen. Thus, $A \cap B$ is a $\mathcal{I}p$-closed set.

□

Definition 3.3. The intersection of all pre-closed sets containing A defines the pre-closure of A, which is denoted as $pcl(A)$.

Theorem 3.8. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space, and let $A \subseteq X$. Then, the following are equivalent*:

1. A is $\mathcal{I}p$-closed.
2. $cl^*(A) \subseteq U$ whenever $A \subseteq U$ and U is preopen in X.
3. For every $x \in cl^*(A)$, $pcl(\{x\}) \cap A \neq \emptyset$.
4. $cl^*(A) - A$ contains no nonempty preclosed set.
5. $A^* - A$ contains no nonempty preclosed set.

Proof.

(1) ⇒ (2). Suppose A is a $\mathcal{I}p$-closed set in X. then $A^* \subseteq U$ whenever $A \subseteq U$ and U is preopen in X and so, $cl^*(A) = A \cup A^* \subseteq U$ whenever $A \subseteq U$ and U is preopen in X.

(2) ⇒ (3). Assume that $x \in cl^*(A)$. If $pcl(\{x\}) \cap A$ = $\emptyset$, then $A \subseteq X - pcl(\{x\})$. By (2), $cl^*(A) \subseteq X - pcl(\{x\})$. This statement is inconsistent with our fact. Hence $pcl(\{x\}) \cap A \neq \emptyset$.

(3) ⇒ (4). Assume that $F \subseteq cl^*(A) - A$, F is preclosed and $x \in F$. Since $F \subseteq X - A$ and F is preclosed, $pcl(\{x\}) \cap A = \emptyset$. Since $x \in cl^*(A)$ by(3), $pcl(\{x\}) \cap A \neq \emptyset$, this is contradiction to our fact. Thus, there are no nonempty preclosed sets in $cl^*(A) - A$.

(4) ⇒ (5). Since $cl^*(A) - A$ is equal to $(A \cup A^*) - A$, which in turn is equal to $(A \cup A^*) \cap A^c$, we have that $(A \cap A^c) \cup (A^* \cap A^c) = A^* \cap A^c = A^* - A$. Thus, $A^* - A$ has no nonempty preclosed set.

(5) ⇒ (1). Suppose that $A \subseteq U$, which is preopen, we have, $X - U \subseteq X - A$. Thus, $A^* \cap (X - U) \subseteq A^* \cap (X - A) = A^* - A$. Since A^* is always closed, it follows that A^* is preclosed. Therefore, $A^* \cap (X - U)$ is a preclosed set that is contained in $A^* - A$. Consequently, $A^* \cap (X - U) = \emptyset$, and so $A^* \subseteq U$. This implies that A is a $\mathcal{I}p$-closed.

□

Proposition 3.3. *Suppose that $(X, \tau, \mathcal{I})$ is an ideal topological space.*

1. *If $A \in \mathcal{I}$, then A is a $\mathcal{I}p$-closed set.*
2. *If $A \subseteq X$, then A^* is a $\mathcal{I}p$-closed set.*

Proof.
1. Assume that $A \in \mathcal{I}$ and $A \subseteq U$, where U is preopen. Since $A^* = \emptyset$ for every $A \in \mathcal{I}$, we have $cl^*(A) = (A \cup A^*) = A$, which is contained in U. Thus, by Theorem 3.8, A is a $\mathcal{I}p$-closed set.
2. Let $A \in (X, \tau, \mathcal{I})$ and suppose that A^* is a subset of the preopen set U. Since $(A^*)^*$ is always a subset of A^*, it follows that $(A^*)^* \subseteq U$ whenever $A^* \subseteq U$ and U is preopen. Thus, A^* is $\mathcal{I}p$-closed.

□

Theorem 3.9. *Suppose $(X, \tau, \mathcal{I})$ is an ideal topological space and A, B are subsets of X such that $A \subseteq B \subseteq cl^*(A)$. If A is a $\mathcal{I}p$-closed set, then B is a $\mathcal{I}p$-closed set.*

Proof. Let A be an $\mathcal{I}p$-closed set. By Theorem 3.8, we know that $cl^*(A) - A$ has no nonempty preclosed set. Since $A \subseteq B \subseteq cl^*(A)$, we have that $cl^*(B) - B \subseteq cl^*(A) - A$ and therefore, $cl^*(B) - B$ has no nonempty preclosed set. By applying Theorem 3.8 once more, we can conclude that B is a $\mathcal{I}p$-closed set. □

Theorem 3.10. *Consider $(X, \tau, \mathcal{I})$ is an ideal topological space, and let A be a subset of X. Then A is $\mathcal{I}p$-closed if and only if it can be expressed as $A = F$-N, where F is a $*$-closed set and $N = A^*$-A.*

Proof. Suppose A is $\mathcal{I}p$-closed, we can apply Theorem 3.8 to conclude that $N = A^* - A$ does not contain any nonempty preclosed set. Let $F = cl^*(A)$, which is $*$-closed, and consider $F - N = cl^*(A) - (A^* - A) = (A \cup A^*) \cap (A^* \cap A^c)^c = (A \cup A^*) \cap ((A^*)^c \cup A) = (A \cup A^*) \cap (A \cup (A^*)^c) = A \cup (A^* \cap (A^*)^c) = A$.

Conversely, suppose that $A = F - N$, where F is $*$-closed and N contain no nonempty preclosed set. Suppose there exists a preopen set U such that $A \subseteq U$. It follows that $F - N \subseteq U$ which means that $F \cap (X - U) \subseteq N$. Since $A \subseteq F$ and $F^* \subseteq F$, we also have $A^* \subseteq F^*$. Therefore, we have $A^* \cap (X - U) \subseteq F^* \cap (X - U) \subseteq F \cap (X - U) \subseteq N$. By the hypothesis that $A^* \cap (X - U)$ is preclosed and contains no nonempty set, we conclude that $A^* \cap (X - U) = \emptyset$. Therefore, $A^* \subseteq U$, which implies that A is $\mathcal{I}p$-closed. □

Theorem 3.11. *In an ideal topological space $(X, \tau, \mathcal{I})$. Assume that A is $\mathcal{I}p$-closed and B is $*$-closed in X, then $A \cap B$ is $\mathcal{I}p$-closed in X.*

Proof. Assume that A and B are subsets of X and that U is a preopen set in X containing $A \cap B$. Then, $A \subseteq U \cup (X - B)$. From the fact that A is $\mathcal{I}p$-closed in X, it follows that either $A^* \subseteq U \cup (X - B)$ or $A^* \cap B \subseteq U$. Then $(A \cap B)^* \subseteq A^* \cap B^* \subseteq A^* \cap B \subseteq U$, since B is $*$-closed. Hence, $A \cap B$ is $\mathcal{I}p$-closed in X. □

Definition 3.4. Consider two ideal topological spaces, denoted by $(X, \tau, \mathcal{I})$ and $(Y, \sigma, \mathcal{J})$. A map $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is called $\mathcal{I}p$-continuous map if $f^{-1}(V)$ is a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$ for every closed set V in $(Y, \sigma, \mathcal{J})$.

Example 3.12. *We can define two ideal topological spaces $(X, \tau, \mathcal{I})$ and $(Y, \sigma, \mathcal{J})$, where $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{c\}, \{b, c\}\}$, $\mathcal{I} = \{\emptyset\}$, $\sigma = \{Y, \emptyset, \{c\}\}$ and $\mathcal{J} = \{\emptyset, \{c\}\}$. Consider the map $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ by assigning $f(a) = b$, $f(b) = a$ and $f(c) = c$. Thus, f is a $\mathcal{I}p$-continuous map.*

Lemma 3.13. *Every continuous map in an ideal topological space is a $\mathcal{I}p$-continuous map.*

Proof. Suppose we have a continuous map $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ between ideal topological spaces, along with a closed set V in Y, Since f is continuous, $f^{-1}(V)$ is closed in X and therefore $\mathcal{I}p$-closed in X, which implies that f is a $\mathcal{I}p$-continuous map. □

It is worth noting that the inverse of the lemma mentioned above does not hold in general. An example that demonstrates this fact is as follows:

Example 3.14. *The sets X and Y are defined as $\{a,b,c\}$, and $\tau = \{X,\emptyset,\{b\}\}$, $\mathcal{I} = \{\emptyset,\{b\}\}$, $\sigma = \{Y,\emptyset,\{b,c\}\}$, and $\mathcal{J} = \{\emptyset\}$. We define a map $f : (X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ by setting $f(a) = c$, $f(b) = a$, $f(c) = b$. Although f is a $\mathcal{I}p$-continuous, it is not continuous, since for a closed set $\{a\}$ in Y, $f^{-1}(\{a\}) = \{b\}$ is not closed in X.*

Lemma 3.15. *Every $\mathcal{I}p$-continuous map in an ideal topological space is a $\mathcal{I}g$-continuous map.*

Proof. Assuming that $f : (X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ is a $\mathcal{I}p$-continuous map between ideal topological spaces, and V is a closed set in Y, Since f is $\mathcal{I}p$-continuous, then we have that $f^{-1}(V)$ is $\mathcal{I}p$-closed in X. Moreover, $f^{-1}(V)$ is also $\mathcal{I}g$-closed in X by Lemma 3.2. Therefore, we can conclude that f is a $\mathcal{I}g$-continuous map. □

It is worth noting that the inverse of the lemma mentioned above does not hold in general. An example that demonstrates this fact is as follows:

Example 3.16. *Let X be the set $\{a,b,c\}$, and let Y be the same set as X. Let $\tau = \{X,\emptyset,\{b,c\}\}$, $\mathcal{I} = \{\emptyset,\{a\}\}$, $\sigma = \{Y,\emptyset,\{a\},\{b,c\}\}$, and $\mathcal{J} = \{\emptyset\}$. The map $f : (X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ by setting $f(a) = b$, $f(b) = a$, $f(c) = c$. Following that, the $\mathcal{I}g$-closed sets are X,$\emptyset$,$\{a\}$,$\{a,b\}$,$\{a,c\}$ and the $\mathcal{I}p$-closed sets are X,$\emptyset$,$\{a\}$. Since for a closed set $\{b,c\}$ in Y, $f^{-1}(\{b,c\}) = \{a,c\}$ is not a $\mathcal{I}p$-closed set in X. As a result, f is a $\mathcal{I}g$-continuous map but not a $\mathcal{I}p$-continuous map.*

Lemma 3.17. *Every $\mathcal{I}p$-continuous map in an ideal topological space is a $\alpha\mathcal{I}g$-continuous map.*

Proof. Assuming that $f : (X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ is a $\mathcal{I}p$-continuous map between ideal topological spaces and V is a closed set in Y. As f is $\mathcal{I}p$-continuous, we can infer that $f^{-1}(V)$ is $\mathcal{I}p$-closed in X. As per Lemma 3.4, $f^{-1}(V)$ is $\alpha\mathcal{I}g$-closed in X. This implies that f is a $\alpha\mathcal{I}g$-continuous map. □

It is worth noting that the inverse of the lemma mentioned above does not hold in general. An example that demonstrates this fact is as follows:

Example 3.18. *Let X be the set $\{a,b,c\}$, and let Y be the same set as X. Let $\tau = \{X,\emptyset,\{b\},\{a,c\}\}$, $\mathcal{I} = \{\emptyset,\{b\}\}$, $\sigma = \{Y,\emptyset,\{a\}\}$, and $\mathcal{J} = \{\emptyset\}$. The map $f : (X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ by setting $f(a) = a$, $f(b) = c$, $f(c) = b$. Following that, the $\alpha\mathcal{I}g$-closed sets are X, $\emptyset$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$ and the $\mathcal{I}p$-closed sets are X, $\emptyset$, $\{b\}$, $\{a,c\}$. Since for a closed set $\{b,c\}$ in Y, $f^{-1}(\{b,c\}) = \{b,c\}$ is not a $\mathcal{I}p$-closed set in X. Therefore, f is a $\alpha\mathcal{I}g$-continuous map but not a $\mathcal{I}p$-continuous map.*

Lemma 3.19. *Every $\mathcal{I}p$-continuous map in an ideal topological space is a $\mathcal{I}rg$-continuous map.*

Proof. Suppose $f : (X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ be a $\mathcal{I}p$-continuous map between the ideal topological spaces, and let V be a closed set in Y. As f is $\mathcal{I}p$-continuous, we know that $f^{-1}(V)$ is a $\mathcal{I}p$-closed set in X with respect to the ideal topology. By applying Lemma 3.6, we conclude that $f^{-1}(V)$ is a $\mathcal{I}rg$-closed set in X. Therefore, f is a $\mathcal{I}rg$-continuous map. □

It is worth noting that the inverse of the lemma mentioned above does not hold in general. An example that demonstrates this fact is as follows:

Example 3.20. *The sets X and Y are defined as $\{a,b,c\}$, $\tau = \{X,\emptyset,\{a\},\{b\},\{a,b\}\}$, $\mathcal{I} = \{\emptyset,\{c\}\}$, $\sigma = \{Y,\emptyset,\{b\},\{a,b\}\}$ and $\mathcal{J} = \{\emptyset\}$. The map $f : (X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ by setting $f(a) = c$, $f(b) = a$, $f(c) = b$. Following that, the $\mathcal{I}rg$-closed sets are X, $\emptyset$, $\{c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$ and the $\mathcal{I}p$-closed sets are X, $\emptyset$, $\{c\}$, $\{a,c\}$, $\{b,c\}$. Since for a closed set $\{a,c\}$ in Y, $f^{-1}(\{a,c\}) = \{a,b\}$ is not a $\mathcal{I}p$-closed set in X. Therefore, f is a $\mathcal{I}rg$-continuous map but not a $\mathcal{I}p$-continuous map.*

Remark 3.2. The composition of two $\mathcal{I}p$-continuous maps may not be $\mathcal{I}p$-continuous in general. An example illustrating this fact is as follows:

Example 3.21. *Suppose that $X = \{a,b,c\}$, be with $\tau = \{X,\emptyset,\{b\},\{c\},\{b,c\}\}$, $\mathcal{I} = \{\emptyset,\{c\}\}$, let $Y = \{a,b,c\}$ be with $\sigma = \{Y,\emptyset,\{a,c\},\{b\}\}$, $\mathcal{J} = \{\emptyset,\{a\}\}$ and let $Z = \{a,b,c\}$ be with $\eta = \{Z,\emptyset,\{b\},\{c\},\{a,c\},\{b,c\}\}$, $\mathcal{K} = \{\emptyset\}$. We can express the mappings of $f : (X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ by setting $f(a) = b$, $f(b) = c$, $f(c) = a$ and $g : (Y,\sigma,\mathcal{J}) \to (Z,\eta,\mathcal{K})$ by setting $g(a) = a$, $g(b) = c$, $g(c) = b$. We can conclude that f and g are $\mathcal{I}p$-continuous maps, but their composition $g \circ f$ is not a $\mathcal{I}p$-continuous map.*

Definition 3.5. Consider two ideal topological spaces, denoted by $(X,\tau,\mathcal{I})$ and $(Y,\sigma,\mathcal{J})$. A map $f : (X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ is called $\mathcal{I}p$-irresolute if $f^{-1}(V)$ is a $\mathcal{I}p$-closed set in $(X,\tau,\mathcal{I})$ for every $\mathcal{I}p$-closed set V in $(Y,\sigma,\mathcal{J})$.

Example 3.22. *Assume that $X = Y = \{a,b,c\}$, $\tau = \{X,\emptyset,\{b\},\{a,b\},\{b,c\}\}$, $\mathcal{I} = \{\emptyset,\{a\},\{b\},\{a,b\}\}$, $\sigma = \{Y,\emptyset,\{a\},\{b\},\{a,b\}\}$ and $\mathcal{J} = \{\emptyset,\{c\}\}$. The map $f : (X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ by setting $f(a) = c$, $f(b) = a$ and $f(c) = b$. Thus, f is a $\mathcal{I}p$-irresolute map.*

Lemma 3.23. *Every $\mathcal{I}p$-irresolute map in an ideal topological space is a $\mathcal{I}p$-continuous map.*

Proof. Assuming $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is a $\mathcal{I}p$-irresolute map and V is a closed set in $(Y, \sigma, \mathcal{J})$, it follows that V is also a $\mathcal{I}p$-closed set. By the definition of $\mathcal{I}p$-irresolute map, we can infer that $f^{-1}(V)$ is a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$, which implies that f is a $\mathcal{I}p$-continuous map. ☐

It is worth noting that the inverse of the lemma mentioned above does not hold in general. An example that demonstrates this fact is as follows:

Example 3.24. *Let X be the set $\{a, b, c\}$, and let Y be the same set as X. Let $\tau = \{X, \emptyset, \{c\}, \{b, c\}\}$, $\mathcal{I} = \{\emptyset\}$, $\sigma = \{Y, \emptyset, \{c\}\}$, and $\mathcal{J} = \{\emptyset, \{c\}\}$. The map $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ by setting $f(a) = b$, $f(b) = a$, $f(c) = c$. Thus, f is a $\mathcal{I}p$-continuous map, but it does not satisfy the conditions for being $\mathcal{I}p$-irresolute*

Definition 3.6. Consider two ideal topological spaces, denoted by $(X, \tau, \mathcal{I})$ and $(Y, \sigma, \mathcal{J})$. A map $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is called strongly $\mathcal{I}p$-continuous map if $f^{-1}(V)$ is a closed set in $(X, \tau, \mathcal{I})$ for every $\mathcal{I}p$-closed set V in $(Y, \sigma, \mathcal{J})$.

Example 3.25. *Assume that $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c\}\}$, $\mathcal{I} = \{\emptyset\}$, $\sigma = \{Y, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$, and $\mathcal{J} = \{\emptyset, \{a\}\}$. Define the map $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ by setting $f(a) = b$, $f(b) = c$ and $f(c) = a$. Thus, f is a strongly $\mathcal{I}p$-continuous map.*

Lemma 3.26. *In an ideal topological space, every continuous map is a strongly $\mathcal{I}p$-continuous map.*

Proof. Suppose $(X, \tau, \mathcal{I})$ and $(Y, \sigma, \mathcal{J})$ are ideal topological spaces, with $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ a continuous map, and let V be a closed set in Y. Then, V is a $\mathcal{I}p$-closed set in Y. Additionally, since f is continuous, $f^{-1}(V)$ is closed in X. Therefore, f is a strongly $\mathcal{I}p$-continuous map. ☐

It is worth noting that the inverse of the lemma mentioned above does not hold in general. An example that demonstrates this fact is as follows:

Example 3.27. *Consider the sets $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{b\}, \{c\}, \{b, c\}\}$, $\mathcal{I} = \{\emptyset\}$, $\sigma = \{Y, \emptyset, \{b\}, \{a, b\}\}$, and $\mathcal{J} = \{\emptyset, \{a\}\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be defined as $f(a) = a$, $f(b) = c$, $f(c) = b$. Then, f is a strongly $\mathcal{I}p$-continuous map but not a continuous map.*

Proposition 3.4. *In an ideal topological space, the composition of two strongly $\mathcal{I}p$-continuous maps is also a strongly $\mathcal{I}p$-continuous map.*

Proof. Suppose $(X, \tau, \mathcal{I})$, $(Y, \sigma, \mathcal{J})$, and $(Z, \eta, \mathcal{K})$ are ideal topological spaces, and let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ and $g : (Y, \sigma, \mathcal{J}) \to (Z, \eta, \mathcal{K})$ be a strongly $\mathcal{I}p$-continuous map. Let V be a closed set in $(Z, \eta, \mathcal{K})$, which is also a $\mathcal{I}p$-closed set in $(Z, \eta, \mathcal{K})$. Since g is a strongly $\mathcal{I}p$-continuous map, $g^{-1}(V)$ is a closed set in $(Y, \sigma, \mathcal{J})$, and hence it is a $\mathcal{I}p$-closed set in $(Y, \sigma, \mathcal{J})$. By the same reasoning, since f is a strongly $\mathcal{I}p$-continuous map, $f^{-1}(g^{-1}(V))$ is a closed set in $(X, \tau, \mathcal{I})$, and therefore, $(g \circ f)^{-1}(V)$ is a closed set in $(X, \tau, \mathcal{I})$. Thus, $g \circ f$ is a strongly $\mathcal{I}p$-continuous map. ☐

Theorem 3.28. *Consider $(X, \tau, \mathcal{I})$, $(Y, \sigma, \mathcal{J})$ and $(Z, \eta, \mathcal{K})$ as ideal topological spaces and let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ and $g : (Y, \sigma, \mathcal{J}) \to (Z, \eta, \mathcal{K})$ be two mappings.*

1. If f is a $\mathcal{I}p$-continuous map and g is a continuous map, then $g \circ f$ is a $\mathcal{I}p$-continuous map.
2. If f is a strongly $\mathcal{I}p$-continuous map and g is a $\mathcal{I}p$-continuous map, then $g \circ f$ is a continuous map.
3. If f and g are strongly $\mathcal{I}p$-continuous maps, then $g \circ f$ is a $\mathcal{I}p$-irresolute map.
4. If f is a strongly $\mathcal{I}p$-continuous map and g is a $\mathcal{I}p$-irresolute map, then $g \circ f$ is a continuous map.
5. If f and g are $\mathcal{I}p$-irresolute maps, then $g \circ f$ is a $\mathcal{I}p$-irresolute map.
6. If f is a $\mathcal{I}p$-irresolute map and g is a continuous map, then $g \circ f$ is a $\mathcal{I}p$-irresolute map.
7. If f is a $\mathcal{I}p$-irresolute map and g is a $\mathcal{I}p$-continuous map, then $g \circ f$ is a $\mathcal{I}p$-continuous map.

Proof.

1. Assuming that V is a closed set in $(Z, \eta, \mathcal{K})$, and since g is a continuous map, we know that $g^{-1}(V)$ is a closed set in $(Y, \sigma, \mathcal{J})$. Furthermore, since f is a $\mathcal{I}p$-continuous map, we have that $f^{-1}(g^{-1}(V))$ is a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$, which implies that $(g \circ f)^{-1}(V)$ is a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$. Consequently, we can conclude that $g \circ f$ is a $\mathcal{I}p$-continuous map.
2. Suppose that V is a closed set in $(Z, \eta, \mathcal{K})$, and g is a $\mathcal{I}p$-continuous map. Thus, $g^{-1}(V)$ is $\mathcal{I}p$-closed in $(Y, \sigma, \mathcal{J})$. Additionally, since f is a strongly $\mathcal{I}p$-continuous map, we have that

$f^{-1}(g^{-1}(V))$ is a closed set in $(X, \tau, \mathcal{I})$, which implies that $(g \circ f)^{-1}(V)$ is a closed set in $(X, \tau, \mathcal{I})$. Therefore, we can conclude that $g \circ f$ is a continuous map.

3. Assuming that V be a closed set in $(Z, \eta, \mathcal{K})$ then it is a $\mathcal{I}p$-closed set in $(Z, \eta, \mathcal{K})$. We have that g is a strongly $\mathcal{I}p$-continuous map, $g^{-1}(V)$ is a closed set in $(Y, \sigma, \mathcal{J})$ then it is a $\mathcal{I}p$-closed set in $(Y, \sigma, \mathcal{J})$. Now as f is a strongly $\mathcal{I}p$-continuous map, we get $f^{-1}(g^{-1}(V))$ is a closed set in $(X, \tau, \mathcal{I})$ then it is a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$. So $(g \circ f)^{-1}(V)$ is a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$, thus $g \circ f$ is a $\mathcal{I}p$-irresolute map.
4. Let V be a closed set in $(Z, \eta, \mathcal{K})$ then it is a $\mathcal{I}p$-closed set in $(Z, \eta, \mathcal{K})$. Since g is a $\mathcal{I}p$-irresolute map, $g^{-1}(V)$ is a $\mathcal{I}p$-closed in $(Y, \sigma, \mathcal{J})$. Now as f is a strongly $\mathcal{I}p$-continuous map, we get $f^{-1}(g^{-1}(V))$ is a closed set in $(X, \tau, \mathcal{I})$. So $(g \circ f)^{-1}(V)$ is a closed set in $(X, \tau, \mathcal{I})$, thus $g \circ f$ is a continuous map.
5. Suppose V is a $\mathcal{I}p$-closed set in $(Z, \eta, \mathcal{K})$. As g is a $\mathcal{I}p$-irresolute map, we have that $g^{-1}(V)$ is a $\mathcal{I}p$-closed set in $(Y, \sigma, \mathcal{J})$. Similarly, since f is also $\mathcal{I}p$-irresolute map, we have that $f^{-1}(g^{-1}(V))$ is a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$, implying that $(g \circ f)^{-1}(V)$ is a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$. Therefore, $g \circ f$ is a $\mathcal{I}p$-irresolute map.
6. Let V be a closed set in $(Z, \eta, \mathcal{K})$. Since g is continuous, $g^{-1}(V)$ is a closed set in $(Y, \sigma, \mathcal{J})$. Since f is a $\mathcal{I}p$-irresolute map, we get $f^{-1}(g^{-1}(V))$ is a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$. That is, $(g \circ f)^{-1}(V)$ is a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$. So $g \circ f$ is a $\mathcal{I}p$-irresolute map.
7. Suppose V is a closed set in $(Z, \eta, \mathcal{K})$. As g is a $\mathcal{I}p$-continuous map, $g^{-1}(V)$ is a $\mathcal{I}p$-closed set in $(Y, \sigma, \mathcal{J})$. But f is a $\mathcal{I}p$-irresolute map, we have $f^{-1}(g^{-1}(V))$ is a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$. Which gives, $(g \circ f)^{-1}(V)$ is a $\mathcal{I}p$-closed set in $(X, \tau, \mathcal{I})$. Therefore, $g \circ f$ is a $\mathcal{I}p$-continuous map.

□

Definition 3.7. Consider two ideal topological spaces, denoted by $(X, \tau, \mathcal{I})$ and $(Y, \sigma, \mathcal{J})$. A map $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is called $\mathcal{I}p$-closed map[22] if the image of every closed set in X is a $\mathcal{I}p$-closed set in Y.

The authors of[22] establish the vital properties of $\mathcal{I}p$-closed maps and their interconnections with other generalized closed maps, including $\mathcal{I}g$-closed maps, $\alpha\mathcal{I}g$-closed maps, and $\mathcal{I}rg$-closed maps in ideal topological spaces.

4 CONCLUSIONS

In this paper, we defined new class of generalized closed sets called $\mathcal{I}p$-closed sets in ideal topological spaces. We looked at the main characteristics of this new class of generalized closed sets and compared it to other classes of generalized closed sets that are already exist in ideal topological spaces. By utilizing these newly found $\mathcal{I}p$-closed sets, we discovered a $\mathcal{I}p$-continuous map, a $\mathcal{I}p$-irresolute map, and a strongly $\mathcal{I}p$-continuous map. In each segment, we took a look at some of their most significant features. In future, we can enrich the concept of generalized $\mathcal{I}p$-closed sets in ideal topological spaces by extending it to fuzzy topological spaces. Overall, the generalized $\mathcal{I}p$-closed set in ideal topological spaces is likely to involve a combination of theoretical developments, applications to other areas of mathematics, and the construction of new examples and counterexamples.

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Authors have declared that no competing interests exist.

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